ROBUST ESTIMATION: TOWARDS AUTOMATED IMAGE ORIENTATION

Norm Campbell and Xiaoliang Wu
CSIRO Mathematical and Information Sciences
65 Brockway Road, Floreat Park, Western Australia 6014
Phone (618) 9333 6162, Fax (618) 9333 6121
Norm.Campbell@csiro.au

Abstract

Image orientation is a process to recover the spatial relationships between images. It is the first and very important step in 3D surface reconstruction, digital elevation model generation and image ortho-rectification from images (multiple stereo pairs). Outliers are very common among the corresponding tie points, which are usually automatically determined using image matching techniques. The tie points are then used to estimate the image orientation parameters. Detecting and removing these outliers from tie points is critical for accurately recovering the spatial relationship between images and for the consequent image processing tasks.

This paper presents a robust estimation method for automated relative orientation parameter estimation. The basic concept of relative orientation is first studied based on the collinearity equations. Then a new image orientation calculation method is proposed, called the conditional reduced-parameter gradient-substitution method (CRPGS). It is shown numerically that the CRPGS method is equivalent to the full-parameter method; however, the CRPGS method can reduce the computation time significantly, especially for large numbers of points. Discussion on optimal estimation for relative orientation parameters is then presented. A robust estimator (S-estimator) is introduced for weighted least-squares solutions of the parameters. Based on the above development, a new robust estimation strategy for automated relative orientation is presented. Finally, the experimental results from various estimation methods applied to a set of real images from a practical application are presented and discussed. The proposed robust estimation strategy gives excellent results in all cases.

1. Introduction

As is well known, outliers are common in photogrammetric measurements and their occurrence will inevitably lead to unreliable results. Large outliers are usually mistakes, such as identifying a wrong control point on images or typing in a wrong coordinate; small outliers may result from mismatched points during automated matching processing.

In the current automated digital photogrammetry era, a large number of observations are often available. For example, relatively few image orientation parameters (normally six exterior orientation parameters per image) are often estimated using hundreds of tie points (compared to using less than 10 points in the early applications), a point in a digital elevation model (DEM) can be interpolated using several matched points around a node, and so on. If outliers can be eliminated from the observations, then greater accuracy can be guaranteed.

In this paper, a relatively new robust approach, called S-estimation, is introduced for image orientation. This paper is organised as follows. Common photogrammetric image orientation approaches are discussed in Section 2. S-estimation is presented in detail in Section 3. Section 4 gives some results from the image orientation tasks and finally, some discussion of this study and other potential applications of robust estimation in photogrammetry are given in Section 5.

2. Image Orientation in Photogrammetry

In photogrammetry, the image orientation parameters are usually found by minimising the sums of squares of residuals between the observed values and their predicted values using the collinearity condition and/or coplanarity condition.

Assume that the camera station S has object space coordinates (X_S, Y_S, Z_S), that A is an object space point with object space coordinates (X, Y, Z), and that the image point of A is a with image plane coordinates (x, y), image space coordinates $(x, y, -f)$ and auxiliary space coordinates (u, v, w). Then the collinearity equations can be expressed as:

$$
\begin{aligned}
x &= x_0 - f\frac{a_1(X - X_S) + b_1(Y - Y_S) + c_1(Z - Z_S)}{a_3(X - X_S) + b_3(Y - Y_S) + c_3(Z - Z_S)} \\
y &= y_0 - f\frac{a_2(X - X_S) + b_2(Y - Y_S) + c_2(Z - Z_S)}{a_3(X - X_S) + b_3(Y - Y_S) + c_3(Z - Z_S)}
\end{aligned}
\tag{2-1}
$$

where x_0, y_0 are the image plane coordinates of the camera principal point and $a_i, b_i, c_i (i = 1,2,3)$ are the nine elements of the image rotation matrix R. The collinearity equations are usually considered as the most important equations in photogrammetry.

In addition to the collinearity equations, the coplanarity condition is widely used for the case of a stereo image orientation. The coplanarity equation for two identical image points of a stereo image pair is:

$$
F = \begin{vmatrix} B_X & B_Y & B_Z \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \end{vmatrix} = 0 \tag{2-2}
$$

$$
\begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ b_{11} & b_{21} & b_{31} \\ c_{11} & c_{21} & c_{31} \end{bmatrix} \begin{bmatrix} x_1 - x_{01} \\ y_1 - y_{01} \\ -f_1 \end{bmatrix}, \quad \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} a_{12} & a_{22} & a_{32} \\ b_{12} & b_{22} & b_{32} \\ c_{12} & c_{22} & c_{32} \end{bmatrix} \begin{bmatrix} x_2 - x_{02} \\ y_2 - y_{02} \\ -f_2 \end{bmatrix}
$$

where the left camera position is set as the origin of the left image auxiliary space coordinate system, and B_X, B_Y, B_Z are the three baseline components. The subscripts 1 and 2 denote the left and right images, respectively.

The collinearity and coplanarity equations form the fundamentals of photogrammetry for the cases of a single image, two images (a stereo pair), and multiple images. Image orientation in photogrammetry, in theory, can be sorted into three kinds of orientation: 1) interior, 2) relative and 3) absolute/exterior orientations. Relative orientation is mainly dealt with in this paper, since outliers usually occur among tie points used for relative orientation. The relative orientation parameters are usually found based on the coplanarity equation in (2-2), however, as shown in this paper, the relative orientation parameters can also be found based on the collinearity equations in (2-1).

Image orientation needs a reasonable number of tie points; these tie points are usually located in the overlapping areas of stereo image pairs and may or may not have object space coordinates (if they do, they are called control points). These tie points can be found either using image matching techniques or manually identified by human operators (control points always require human operators for identification).

The common estimation technique used in photogrammetry is least squares. The image orientation parameters and/or object space coordinates are usually found by minimising the residual sum of squares (2-3) for the collinearity equations in (2-1) because the three points (camera station S, object space point A and its image point a) are not always collinear:

$$s^2 = \sum (x_i - \hat{x}_i)^2 + \sum (y_i - \hat{y}_i)^2 = \sum r_{xi}^2 + \sum r_{yi}^2 \qquad (2\text{-}3)$$

where x_i, y_i are the measured image plane coordinates of the control points and/or tie points, $\hat{x}_i, \hat{y}_i$ are the fitted image plane coordinates of the control points and/or tie points after estimating the image orientation, and the subscript i represents either the left or right image (or an arbitrary image if multiple images are involved).

It is common practice in the photogrammetric literature to estimate the parameters by linearising the equations using a Taylor expansion (see eg Kraus, 1993, Appendix 4.2-1); this is equivalent to Fisher's scoring algorithm.

There are two sets of unknown parameters in the collinearity equations: the exterior orientation parameters $(X_S, Y_S, Z_S, \varphi, \omega, \kappa)$ or E for each image; and three space coordinates (X, Y, Z) or G for each tie point; E and G can be estimated simultaneously, or we can alternate the estimation. An alternating approach developed by the authors is presented in Section 2.3.

2.1 Estimating two sets of parameters simultaneously (FP approach)

One of the advantages of this approach is that all parameters are solved simultaneously. However, it can become very computationally expensive when there are many tie points (assuming there are *m* images and *n* tie points, the order of the matrix to be inverted is *6m+3n*). This approach is referred to here as FP (Full Parameter) estimation.

2.2 Estimating two sets of parameters alternatively (CRP approach)

The two sets of parameters E and G in the collinearity equations can also be determined as follows: one set of parameters is fixed and treated as known while the other set of parameters is estimated, then in the next iteration, reverse the order to estimate the other set of parameters. This approach is referred to here as conditional reduced-parameter estimation (CRP).

2.3 Estimating two sets of parameters efficiently (CRPGS approach)

In many photogrammetric systems, both approaches (FP and CRP) are commonly used to solve for the image orientation parameters and/or the object space coordinates of the tie points. Initial values are required for all unknown parameters in both approaches. In some situations, especially in close-range photogrammetry applications, it is very difficult to estimate the initial values for both the image orientation parameters and the tie-point object-space coordinates. Given the orientation parameters of the images, the object space coordinates of a tie point can be calculated explicitly from the collinearity equations. A new approach, referred to here as conditional reduced-parameter gradient-substitution (CRPCS), has been developed to estimate the image orientation parameters very efficiently, by modifying the gradients to adjust for the tie-point estimation. The following paragraphs briefly describe the CRPCS approach. A more comprehensive derivation can be obtained by contacting the authors.

The following two formulations describe the collinear relationships of an object point, its corresponding image points and the camera centres in two images, respectively:

$$\begin{aligned} \frac{u_1}{X - X_{S1}} = \frac{v_1}{Y - Y_{S1}} = \frac{w_1}{Z - Z_{S1}} \\ \frac{u_2}{X - X_{S2}} = \frac{v_2}{Y - Y_{S2}} = \frac{w_2}{Z - Z_{S2}} \end{aligned} \qquad (2\text{-}4)$$

By rearranging the above equations, the following equation can be derived:

$$\begin{bmatrix} 1 & 0 & -u_1/w_1 \\ 0 & 1 & -v_1/w_1 \\ 1 & 0 & -u_2/w_2 \\ 0 & 1 & -v_2/w_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{S1} - Z_{S1}(u_1/w_1) \\ Y_{S1} - Z_{S1}(v_1/w_1) \\ X_{S2} - Z_{S2}(u_2/w_2) \\ Y_{S2} - Z_{S2}(v_2/w_2) \end{bmatrix} \qquad (2\text{-}5)$$

Therefore, given the orientation parameters of two images and the tie-point image coordinates, the object space coordinates of a tie point can be calculated explicitly from the collinearity equations:

$$G = Q^{-1}h$$

$$G = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, Q = \begin{bmatrix} 2 & 0 & -\frac{u_1}{w_1} - \frac{u_2}{w_2} \\ 0 & 2 & -\frac{u_1}{w_1} - \frac{u_2}{w_2} \\ -\frac{u_1}{w_1} - \frac{u_2}{w_2} & -\frac{u_1}{w_1} - \frac{u_2}{w_2} & \frac{u_1^2+v_1^2}{w_1^2} + \frac{u_2^2+v_2^2}{w_2^2} \end{bmatrix}$$

$$\begin{bmatrix} h1 \\ h2 \\ h3 \end{bmatrix} = \begin{bmatrix} X_{S1} + X_{S1} - Z_{S1}\frac{u_1}{w_1} - Z_{S2}\frac{u_2}{w_2} \\ Y_{S1} + Y_{S1} - Z_{S1}\frac{v_1}{w_1} - Z_{S2}\frac{v_2}{w_2} \\ Z_{S1}\frac{u_1^2}{w_1^2} - X_{S1}\frac{u_1}{w_1} + Z_{S1}\frac{v_1^2}{w_1^2} - Y_{S1}\frac{v_1}{w_1} + Z_{S2}\frac{u_2^2}{w_2^2} - X_{S2}\frac{u_2}{w_2} + Z_{S2}\frac{v_2^2}{w_2^2} - Y_{S2}\frac{v_2}{w_2} \end{bmatrix}$$

(2-6)

This solution indicates two critical aspects: 1) estimates of the tie-point object space coordinates are calculated explicitly and 2) it establishes a relationship between the image plane coordinates and the image orientation parameters, without directly introducing the tie-point object-space coordinates into the relationship.

If E denotes one of the 18 interior and exterior orientation parameters of the two images $(X_{S1}, Y_{S1}, Z_{S1}, \varphi_1, \omega_1, \kappa_1, f_1, x_{01}, y_{01}, X_{S2}, Y_{S2}, Z_{S2}, \varphi_2, \omega_2, \kappa_2, f_2, x_{02}, y_{02})$, then the first-order partial derivatives of G with respect to E is:

$$\frac{\partial G}{\partial E} = -Q^{-1}\frac{\partial Q}{\partial E}Q^{-1}h + Q^{-1}\frac{\partial h}{\partial E} = Q^{-1}(\frac{\partial h}{\partial E} - \frac{\partial Q}{\partial E}Q^{-1}h) \quad (2\text{-}7)$$

With some mathematical manipulation, the partial derivatives of x, y (image plane coordinates) with respect to $X_S, Y_S, Z_S, \varphi, \omega, \kappa, f, x_0, y_0$ (image orientation parameters) in the collinearity equations can be derived explicitly; this allows the image orientation parameters to be estimated efficiently without solving for the object space coordinates simultaneously.

The CRPCS approach has the following advantages over the CRP and FP approaches: 1) only small matrices need to be inverted, regardless of the numbers of tie points; 2) the object space coordinates can be calculated explicitly; 3) it is suitable for both relative and absolute orientation calculations; and 4) it is relatively straightforward to implement, and offers the same accuracy and rate of convergence as the FP approach.

3 Robust Estimation

A theory of robustness has been developed in the statistical literature based on the pioneering work of Huber (1972) and Hampel (see references in Hampel *et al.*, 1986). Robust estimation determines solutions which provide sensible estimates for the main body of data, by down-weighting those observations which are considered to be atypical.

One of the difficulties with calculating useful robust estimates is that the final result can depend on the initial estimates. A common implementation, often referred to as w-estimation, is: i) calculate some initial estimates of means and (co)variances; ii) determine the individual residuals or Mahalanobis distances of the observations from these means, relative to the (co)variances; iii) calculate weights which are related inversely to the magnitudes of the residuals or Mahalanobis distances; and iv) calculate weighted means and (co)variances. There are two common forms of weight function: one in which the influence of an observation on the mean increases linearly for an observation which belongs to the main body of data, and then remains constant; and one in which the influence of an observation is zero for very discrepant observations.

In an ideal approach, the resulting means and (co)variances will relate only to the main body of data, and are not affected by the outliers. Rousseeuw (1986) introduced a high-breakdown estimator of covariance, based on minimising the volume of the ellipsoid based on roughly half the data. He takes so-called elemental samples based on subsets of observations, calculates the means and covariances, and scales the corresponding determinant by an appropriate quantile of the corresponding Mahalanobis distances; he repeats this for a large number of elemental samples. The robust estimates are the means and covariances corresponding to the minimum determinant over the elemental samples.

S-estimators have been proposed as a generalisation of the minimum volume ellipsoid procedure (see for example, Lopuhaa and Rousseeuw, 1991). The determinant of the covariance matrix is again minimised, this time subject to a constraint on the magnitude of the corresponding Mahalanobis distances. Campbell *et al.* (1998) proposed combining the elemental sampling suggested for the minimum volume ellipsoid solution with the w-estimator equations arising from the S-estimator solution, to obtain the constrained minimum determinant from much fewer elemental samples.

3.1 Estimating Image Orientation Parameters using S-estimators

S-estimators minimise the residual sum of squares arising from (2-1), subject to a penalty on the size of the scaled residuals. Formally, this can be described as minimising $\log s^2$ subject to

$$n^{-1}\sum_{i=1}^{n}\{\rho(r_{xi}/s)+\rho(r_{yi}/s)\}=b_0 \qquad (3\text{-}1)$$

where s^2 denotes the residual sum of squares in (2-3); r_{xi} and r_{yi} are the residuals for the ith observation; ρ is the kernel of a 'robust density', which is assumed to be symmetric with continuous derivative $\psi(.)$, and $\rho(0)=0$; and $b_0=E(\rho)$, where $E(\rho)$ is the expected value of ρ (under the assumption of an underlying Gaussian distribution). The choice of ρ and hence ψ is discussed below.

The solution is to calculate a weighted residual sum of squares:

$$s^2=\sum_{i=1}^{n}w(r^*_{xi})r^2_{xi}/\sum_{i=1}^{n}\psi(r^*_{xi})r^*_{xi}+\sum_{i=1}^{n}w(r^*_{yi})r^2_{yi}/\sum_{i=1}^{n}\psi(r^*_{yi})r^*_{yi} \qquad (3\text{-}2)$$

where

$$
\begin{aligned}
& r_i^* = r_i / s \\
& \psi(r_i^*) = \rho'(r_i^*) \\
& w(r_i^*) = \psi(r_i^*) / r_i^*
\end{aligned}
\qquad (3\text{-}3)
$$

and $w(r_i^*)$ is a weight function.

The particular form of the rho (ρ) function used here is Tukey's biweight function, where the first term is the kernel of a Gaussian density, and the remaining terms provide protection against atypical observations (see, eg, Campbell *et al.*, 1998, p.2687). The corresponding (ψ) function is a redescending influence function in which (absolute) residuals greater than a cut-off have zero influence.

The implemented S-estimator procedure for image orientation is as follows:

1. select a so-called elemental subset of the observations, at random – the number of observations in each elemental subset should be just sufficient to estimate the parameters
2. estimate the orientation parameters using the usual calculations
3. calculate the residuals for all of the observations, and scale the residuals iteratively until the constraint on the $\rho(r_i^*)$ in (3-1) is satisfied,
4. calculate the weights $w(r_i^*)$ in (3-3) and carry out a weighted fit to estimate the orientation parameters. In practice, the weighted fit is equivalent to an iterative weighted least squares fit. This step is repeated until convergence, and,
5. return to step 1 to select another elemental subset and repeat steps 2-4.

3.2 Comments on the S-estimator solution

S-estimation can be considered as an amalgamation of two approaches to robustness. The first is the weighted M-estimator (or w-estimator) calculations; the second is the generation of elemental subsets for the least median of squares calculations (Rousseeuw and Yohai, 1984, Hampel *et al.*, 1986, p.330). There are three critical aspects to the S-estimation approach proposed here: i) the generation of the elemental subsets; ii) the scaling of the residuals; and iii) the iterative weighted calculations.

The generation of elemental subsets is designed to produce one or more subsets which are free of atypical observations, so that the parameter estimates are roughly what would be expected if atypical observations did not exist.

The scaling of the residuals is a key step. Since the influence function bounds the size of a residual, it has the effect of downweighting the contribution of outliers. When some observations are downweighted, the residuals are increased collectively so that their weighted average will still be equal to that which would be expected if there were no outliers.

The implementation of the iterative weighted calculations which arise as the solution to the formal constrained minimisation problem leads to a considerable reduction in the number of elemental subsets which need to be generated. Experience with using the approach in a number of contexts indicates that the weighted calculations will converge to a ‘robust’ solution provided only that the elemental subset downweights the outliers, and that this sometimes happens even when the elemental subset contains outlier(s).

4 Trial Results of Orientation Parameter Estimation

The robust S-estimator approach was implemented with the image orientation approaches CRP and CRPGS (see Section 2). Since the results for FP and CRPGS are identical (see below), the S-estimator approach was not implemented for the FP solution.

Because the main objective of the experiments is to examine the capabilities for outlier detection, and most outliers occur amongst tie points during the relative orientation stage, the image orientation experiments are concentrated on relative orientation only. The performance of S-estimators for outlier detection for exterior orientation, where errors can occur in both the object and image coordinates, is currently being studied.

4.1 Test images

Aerial video images were chosen to test the various image orientation algorithms. The images were captured using a digital video camera with a focal length of 17mm (MS3100 from DuncanTech) from a light aircraft at roughly 2000-metre height. All images are the same size (1392 lines by 1039 pixels), the pixel size is 0.00465mm by 0.00465mm, and the standard channel model (RGB) was chosen. The overlapping areas among images vary from 90 percent to 50 percent. Figure 4-1 shows the stereo image pair chosen for all image orientation experiments; some automatically matched tie points are also displayed on these images.

4.2 Comparison among CRPGS, FP and CRP and approaches

An experiment was conducted in order to compare the CRPGS and FP approaches. All 71 tie points used in this experiment were manually checked to guarantee that there are no outliers. With the same tie-point image coordinates and the initial orientation parameters set to zero (the baseline is set to 100), the successive residual sums of squares (SSQ) function values and estimated relative orientation parameters are given in Tables 4-1 and 4-2 for the CRPGS and FP approaches, respectively.

Figure 4-1: The stereo image pair chosen for image orientation experiments. A total of 317 tie points were automatically matched and are shown.

Table 4-1: Image orientation results using CRPGS approach (71 tie points used).

Iterations	Sums of Squares	ΔB_Y	ΔB_Z	$\Delta\omega$	$\Delta\varphi$	$\Delta\kappa$
1	0.9695099636	-26.191	-5.053	0.00108	0.00428	0.00310
2	0.0025445172	-13.246	-2.280	-0.00330	0.00592	0.00448
3	0.0005493968	-4.344	-2.927	-0.00720	0.00700	0.00475
4	0.0000925498	-2.535	-3.017	-0.00808	0.00739	0.00476
5	0.0000008165	-2.470	-3.019	-0.00811	0.00741	0.00476
6	0.0000003039	-2.470	-3.019	-0.00811	0.00741	0.00476

Table 4-2: Image orientation results using FP approach (71 tie points used).

Iterations	Sums of Squares	ΔB_Y	ΔB_Z	$\Delta\omega$	$\Delta\varphi$	$\Delta\kappa$
1	0.9695099636	-2.777	-3.666	-0.00811	0.00769	0.00468
2	0.0001942715	-2.484	-3.023	-0.00810	0.00769	0.00475
3	0.0000003513	-2.462	-3.024	-0.00811	0.00739	0.00476
4	0.0000003241	-2.469	-3.019	-0.00811	0.00740	0.00476
5	0.0000003040	-2.469	-3.019	-0.00811	0.00740	0.00476

This and other experiments indicate that the CRPGS approach gives virtually identical results to the FP approach in terms of accuracy and rate of convergence. As mentioned previously, the CRPGS approach doesn't involve large matrix inversions and therefore uses considerably less computation.

The CRP approach was also tested with the same tie points. The successive residual sums of squares (SSQ) function values and estimated relative orientation parameters are given in Table 4-3. The CRP approach converges to the same solution but much more slowly. This behaviour has been observed on a number of data sets.

Table 4-3: Image orientation results using CRP approach (71 tie points used).

Iterations	Sum of Squares	ΔB_Y	ΔB_Z	$\Delta\omega$	$\Delta\varphi$	$\Delta\kappa$
1	0.9695099636	-4.076	-0.603	-0.00302	0.00120	0.00095
10	0.0000786060	-4.526	-3.040	-0.00746	0.00189	0.00392
100	0.0000174551	-1.755	-3.163	-0.00852	0.00192	0.00495
1000	0.0000165127	-1.775	-3.157	-0.00851	0.00208	0.00494
5000	0.0000129435	-1.858	-3.135	-0.00847	0.00270	0.00492
50000	0.0000012021	-2.308	-3.042	-0.00821	0.00615	0.00480
100000	0.0000003559	-2.431	-3.024	-0.00813	0.00711	0.00477
150000	0.0000003065	-2.462	-3.020	-0.00812	0.00734	0.00476
200000	0.0000003040	-2.469	-3.019	-0.00811	0.00740	0.00476

4.3 Robust estimation

In this section, various tie point sets were automatically extracted for the stereo pair in Figure 4-1. The extraction method is a kind of feature-based matching approach. The detailed extraction method is beyond the scope of this paper. Seven data sets were separately extracted using image matching techniques, the numbers of tie points in each data set varying from only 17 to 994. Outliers exist in these tie points.

Table 4-4 shows the performance of S-estimation for image relative orientation estimation. For each set of tie points, the robust approach found the outlier-free solution (the actual orientation parameters are not shown in the Table).

Table 4-4: Results from robust S-estimation for image relative orientation. (total randomly selected elemental sets are 50, the number to the left of /50 is the number of elemental sets out of 50 which successfully found the correct image relative orientation parameters)

Data sets	1	2	3	4	5	6	7
Tie point numbers	17	75	182	317	501	728	994
S-estimation	2/50	4/50	49/50	50/50	50/50	50/50	50/50

5. Discussion

The FP approach gives virtually identical results to the CRPGS approach, but the CRPGS approach is much faster computationally.

In practice, relatively few elemental subsets (of the order of 15-50) seem to be needed to provide the robust solution. This is because the weighted iterative calculations effectively improve upon the initial solution from an elemental subset. Provided an atypical observation is downweighted using the solution from at least one elemental subset, then a robust solution will result.

Acknowledgments

The authors would like to thank HyVista Pty Ltd (Australia) for providing the aerial video images.

References

Campbell, N.A., Lopuhaa, H.P. and Rousseeuw, P.J. (1998). On the calculation of a robust S-estimator of a covariance matrix, *Statistics in Medicine*, 17, 2685-2695.

Hampel, F.R., Ronchetti, E.M., Rousseeuw, P.J. and Stahel, W.A. (1986). *Robust Statistics: The Approach Based on Influence Functions*, John Wiley, New York, pp.502.

Huber, P.J. (1972). Robust statistics: a review, *Annals of Mathematical Statistics*, 43, 1041-1067.

Kraus, K (1993). *Photogrammetry, Volume 1: Fundamentals and Standard Processes,* Dummler, Bonn, pp.397.

Lopuhaa, H.P. and Rousseeuw, P.J. (1991). Breakdown properties of affine equivariant estimators of multivariate location and covariance matrices, *Annals of Statistics*, 19, 229-248.

Rousseeuw, P.J. (1986). Multivariate estimation with high breakdown point, in Grossman, W., Pflug, G., Vince, I. and Wertz, W. (eds), *Mathematical Statistics and Applications*, Reidel Publishing Company, Dordrecht, 283-297.

Rousseeuw, P.J. and Yohai, V. (1984). *Robust Regression by means of S-estimators*, In Robust and Nonlinear Time Series Analysis, J. Franke, W. Hardle and R. D. Martin (eds), Lecture Notes in Statistics 26, Springer, New York, 256-272.