MINIMAL SET OF MARKERS FOR THE WATERSHED TRANSFORM

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Abstract This paper discusses the inverse problem of the watershed. From a partition obtained by the watershed from markers, find the minimal set of markers that reproduces the same watershed partition. We present a solution based on the minimum spanning tree and introduce the concept of marker receptive region. We also review the watershed transform based on the minimum spanning forest problem. Two applications of this problem are presented: aid in assisted still image watershed segmentation, and evaluation of pre-processing filters.

Keywords: Watershed, Minimum Spanning Tree, Minimum Spanning Forest, Image Segmentation.

1. Introduction

The watershed transform is one of the most powerful tools for image segmentation. Although this transform was first introduced more than twenty years ago [3], its study still calls the attention of the scientific community. The watershed from markers, introduced in [8] made the watershed a general approach for image partitioning, often called the Meyer-Beucher paradigm. The watershed from markers is one of the few techniques suitable for interactive segmentation.

Several properties have been studied about the optimality of the watershed partition [5]. One of the best properties of the watershed from markers is regarding the robustness of the marker placement. The watershed transform gives the same results for two set of markers as long as the second contains the first and any marker of the second set is placed inside the same catchment basin obtained from the first set. This property holds if the input image does not contain any plateau. The question addressed by this paper is how to decrease the size of the markers in such a way that the same watershed partition is obtained.

The problem of finding a set of minimal markers for the watershed is modeled using the graph theory, based on the result that the watershed is the solution of the shortest path forest problem [5], which is equivalent to the minimum spanning forest and can be computed on a minimum spanning tree [6]. Using

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this model we arrived at the concept of "marker receptive regions". These regions are the places where at least a marker is required to achieve the same watershed segmentation. It is possible then to include a further constraint to find the best point inside the receptive regions, depending on the application.

Finding the minimal set of markers that reproduces a previous watershed partitioning is useful in several situations. For interactive segmentation, the user has to place foreground markers on the object and background markers outside the object. After the user arrived at a desirable partition, it is useful to know which markers are not necessary to achieve that segmentation. One can assess pre-processing filters by comparing the minimal set of markers required for a particular partitioning.

This paper is organized as follows. Section 2 reviews the watershed transform and its variations under the theory of the minimum spanning forest. Section 3 presents the receptive marker region and the minimal set of markers concepts and techniques to find them. Section 4 shows some applications to highlight the importance of the minimal set of markers and finally, Section 5 presents final comments and conclusions.

2. Watershed review

The watershed transform is a morphological segmentation tool that partitions an image. There are many watershed variations in the literature which makes the subject quite difficult to understand. This brief review of the several watershed transformations and the next section are an attempt to unify the theory and clarify things.

2.1 Basic Definitions and Concepts

Several concepts associated to the watershed are easier to understand if one models a gray scale image as a topographic surface. *Flat zones* of an image are the largest connected components with constant gray scale. They can be of any size, from a single pixel up to the entire image. A *regional minimum* is a flat zone surrounded by flat zones with strictly higher gray scale values. It is impossible to reach a point from the regional minimum to another point with a lower gray level without climbing. Suppose now that a progressive flooding occurs at a hole made at a regional minima. As the water rises from this minima, the flooding will expand creating a lake with depth, surface area and volume. It may happen that the lake invades neighboring basins as the water level increases. There will be a *maximum lake* before the flooding starts invading a basin with a lower regional minima. The *dynamic* [4] of a regional minimum is the depth of its maximum lake. It gives the minimum height a point in the minima has to climb to reach a lower regional minimum. Fig. 1a shows a illustration of the maximum lake and the dynamic of one regional minima in a signal. Similarly to the dynamic measurement, the area and the volume of the maximum lake are called *vanishing area* and *vanishing volume*, respectively. Generically, Meyer and Vachier [10] have called these measurements by *extinguishing values*.

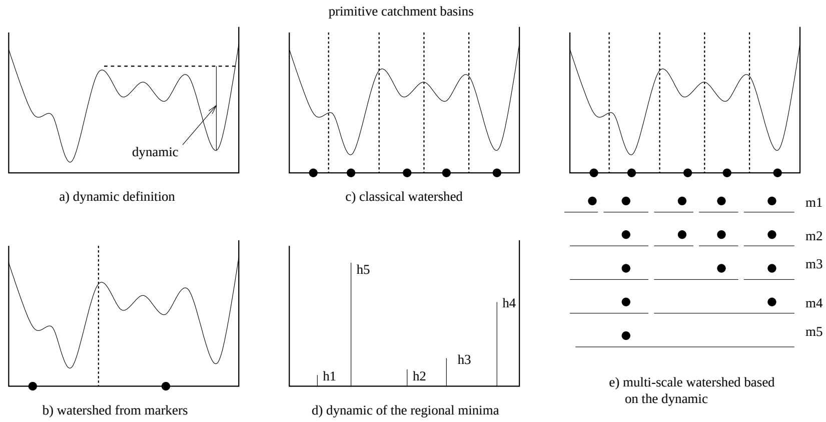


Figure 1. Watershed basic concepts.

2.2 Watershed variations

The partition returned by the watershed transform is usually represented in two ways: by the partition classes, called *catchment basins* (CBs), or by the division lines, called *watershed lines.*

A *marker* is a region involved in the watershed transform. A marker does not need to be connected and it can be a single point or a set of connected or disconnected points. Each marker placed in the image will grow in the watershed transform to generate a catchment basin in the final segmentation. There are three variations of watershed related to the marker concept: the classical watershed, the watershed-from-markers, and the hierarchical watershed.

The *watershed from markers* can be described by flooding a topographical relief model of the input gray-scale image (normally a gradient image). The markers are holes in the image relief where colored water can enter as the relief is flooded. There is one color associated to each marker. As the relief is uniformly flooded, different colored water may meet but cannot be mixed. When all the relief is flooded, each colored water lake defines a catchment basin (CB) associated to the marker. Fig. 1b shows an illustration of a watershed from markers of a signal. The two markers are depicted by black dots.

In the *classical watershed*, there is no marker explicitly given, but it can be described as a particular case of the watershed from markers transform, where the markers are the regional minima of the input image. The catchment basins of the classical watershed are called *primitive catchment basins.* Fig 1c illustrates the classical watershed applied to a signal. This was the first watershed transform reported in the literature and many researchers prefer to keep the watershed name to this definition. This watershed usually exhibits over segmentation when applied to the gradient of a real-life image.

Hierarchical or multi-scale watershed (MWS) transform creates a set of nested partitions. The notion of hierarchical watershed was originally developed by Beucher, reported in [2], but the multi-scale watershed transform was consolidated by the concepts introduced in [6] and [9]. This multi-scale watershed can be obtained by applying the watershed from markers to a decreasing set of markers. These markers are not required explicitly but it helps in describing the hierarchical watershed as special cases of the watershed from markers. The watershed at scale s, $MWS(s)$ is the partition where the markers are the regional minima with dynamic greater or equal s. The watershed at scale 1 (finest partitioning) is the classical watershed, made of the primitive catchment basins. As the scale increases, less markers are involved and the coarsest partition is the entire image obtained from a single marker with largest dynamic. The multi-scale watershed transform is normally obtained using the dynamic concept but it is possible to compute the variations of the watershed using area and volume extinguishing values. Fig. 1e illustrates the multi-scale watershed of a signal where the dynamic of the regional minima are illustrated in Fig 1d.

2.3 Watershed Graph Modeling

Meyer was the first to model the watershed transform using the graph theory framework, but using two distinct formulations for the watershed just presented [7] and the watershed on the neighborhood graph [6], which Meyer calls generically by morphological segmentation. To obtain the neighborhood graph, first the image is partitioned in primitive catchment basins by the classical watershed. Each node represents a catchment basin and an arc exists between two nodes if their corresponding catchment basins are neighbors.

2.4 Graph Definition and Notation

A *graph* $G = (V, A)$ is composed of two sets V and A. V is the set of *nodes*, and A is the set of *arcs* $(p, q), p, q \in V$ associated to a pair of adjacent nodes. In a weighted graph, a weight $w(p, q)$ is associated to each arc. A *path* from v_1 to v_n is a list of unique adjacent nodes $(v_1, v_2, \ldots, v_n)$, $(v_i, v_{i+1}) \in A$. The *path Cost* $C(v_1, v_2, \ldots, v_n)$ is given by a function of the arc weights in the path. The cost between two nodes $C^*(p, q)$ is given by the smallest cost of all the paths between p and q.

Two nodes p and q are *connected* if there is at least one path between p and q. A *connected graph* is a graph where all pair of nodes are connected. A graph has a *cycle* if it has a path from p to q and an arc (q, p) not in the path. A *tree* is a connected graph with no cycles. A *forest* is a collection of trees.

A *spanning tree* of the graph G is a tree that contains all of the nodes of G. The *Minimum Spanning Tree (MST)* of a connected undirected non-negative weighted graph is a spanning tree where the sum of the arc weights is minimal. A *Minimum Spanning Forest (MSF)* of a connected undirected non-negative weighted graph is the following. Given a subset of nodes M, find a forest with minimum total weight such that each node in the complement of M is connected to one and only one node of M.

2.5 Image Forest Transform

The *Image Forest Transform* (IFT) is a concept introduced by Falcao and Lotufo [1] which models an image as a graph. Given a set of marker nodes, the *Shortest-Path Forest* problem framework is a solution to many morphological image processing techniques such as distance transform, morphological reconstruction, strongest edge delineation, and watershed. The Shortest-Path Forest problem finds a forest such that each tree has only one marker and each node belongs to a tree where the path cost from its marker and any node in the tree is minimum. There are three results associated to IFT: the partition, the path and the path cost.

2.6 The watershed in the IFT framework

In most graph shortest path problems, the path cost is given by the sum of the arc weights in the path. The flooding simulation of the watershed corresponds to the solution of shortest path problem in a graph when the path cost is given by the **maximum** of the arc weights in the path. There are two important consequences of using this uncommon case of path cost formulation. The first is that the Shortest Path Forest is equivalent to the Minimum Spanning Forest. Before stating the second consequence, we illustrate the MST, MSF and watershed of a graph by an example shown in Fig. 2. A connected undirected weighted graph has 12 nodes from A to L and 17 arcs of different weights just to simplify the example. The MST is depicted by the bold arcs. The MST total weight is 89 which is the minimum weight for a tree connecting all nodes. If we choose nodes A and D as markers, the MSF will be made of two trees: the tree associated with node A: A, B, E, F, I, J, and K, with weight 31, and the tree associated with node D: C, D, G, H, and L, with weight 53. The total weight of the MSF is 84. The two trees are the catchment basins of the watershed with markers A and D.

When viewing the watershed as a minimum spanning forest, the watershed partition is the forest, a catchment basin is a tree of the MSF and the number of regions of the watershed is the number of trees in the MSF.

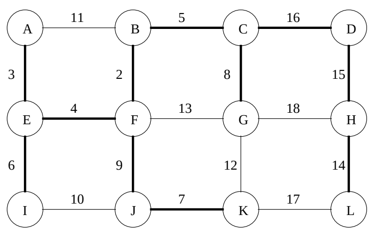


Figure 2. Minimum Spanning Tree.

The second consequence of using the path cost as the maximum arc weight in the path is that the Equation 1 that assigns nodes to the catchment basins based on the shortest path cost must be interpreted with care, in the view of forest partition.

$$CB_k = \{p | C(L_k, p) \leq C(L_j, p), k \neq j\} \quad (1)$$

where CB_k and L_k are the catchment basin k and its marker, respectively. $C(L_k, p)$ is the path cost from the marker L_p to a node p.

Recall the example illustrated in Fig. 2. Suppose that the markers are the nodes A and F. In this case the tree of the MSF associated to the marker A are the nodes A, E, and I. The path cost from marker A to I and from marker F to I are both 6. If the Eq. 1 were not associated to a graph partition, node I could be associated to either tree. In the view of graph partition, the only solution is to assign node I to the marker A, because it must take in account the assignment of node E first. As node E is assigned to the marker A, the path from node I to the marker F passing through node E is not possible.

Under the IFT framework, we can classify the variations of the watershed transforms published in the literature. We propose to call the watershed transform to any transformation that partitions a graph associated to an image into its minimum spanning forest where the path cost is the maximum arc weight in the path. This is the essence of why we cannot classify many region growing algorithms and some watershed extensions reported in the literature as a watershed transform as they do not have the optimality characteristic of the minimum spanning forest.

There are many ways to create a graph and arc weights from an image. They can be classified in two cases: the first is when each node represents a pixel of the image, and the second, when it represents a primitive catchment basin.

One of the simplest variation of the first case is when the arc weight is given by the absolute difference of their gray levels. This is the watershed by dissimilarity reported in [5]. If the arc weight between two neighboring pixels, is assigned the maximum of their values, the minimum spanning forest is the solution of the watershed from markers, and by consequence, to the classical watershed.

In the second case, the graph is called neighborhood graph and the arc weights can be assigned to: the lowest pass found on the common boundary between their catchment basins; by the minimum dynamic (or other extinguishing values) of the regional minima associated to their catchment basins. The first case is equivalent to the classical watershed and the second case, to the hierarchical watershed reported in [9].

The multi-scale watershed is also modeled by this framework as in [6]. Given a minimum spanning tree, if the weights above or equal s are cut, the resulting forest is the watershed at scale s. In the example illustrated in the MST of Fig. 2 the multi-scale watershed at scale 16 creates a forest with two trees: nodes $\{A, B, C, E, F, I, J, K\}$ and nodes $\{D, G, H, L\}$, at scale 2 creates a forest where each node is a tree (finest partition).

3. Minimum markers

The minimum markers problem is the following. For a partition obtained by the watershed transform, find the minimal set of markers to recover the same watershed partition. The main motivation of such problem is to eliminate redundant markers to obtain a goal watershed partitioning.

This problem can be suitably solved using the graph watershed modeling described in the previous sections. In the graph modeling, the problem can be restated as: given a minimum spanning forest obtained from a set of markers, find a minimal set of markers that results in the same spanning forest.

Let us call the goal partition (forest) that we want to reproduce by the set of trees (catchment basins) $CB_i, i = 1, 2, \ldots, n$, with n as the number of regions (trees) in the partition. Any node p is associated to a catchment basin $CB(p)$, that we call here *catchment basin label*.

Frontier nodes of a catchment basin are the nodes adjacent to different catchment basins:

$$F(CB) = \{p \in CB | N(p) \cap CB_i \neq \emptyset, CB \neq CB_i\}$$

where $N(p)$ are the set of nodes adjacent to p.

Strength of a frontier node p is the minimum cost connecting any other catchment basin to p. As the graph is a tree as we are processing the MST, this strength is the smallest weights of the arcs connecting the frontier node to other catchment basins.

$$S(p) = \min\{w(p, r), r \in N(p), r \notin CB(p)\}$$

A trivial solution to recover the partition is to place one marker at each frontier node, labeled with the catchment basin label. We can extend this concept of frontier node to *receptive region*. This region, associated to a frontier node, are the nodes within its catchment basin in which the cost from these nodes to the frontier node is less than the strength of the frontier node.

$$SR(p) = \{q \in CB(p) | C^*(p, q) < S(p)\}$$

To recover the watershed partition, one needs to place at least one marker in each receptive region. There is a simplification where some of these regions do not need to be used. This is the case if two receptive regions intersect. In this case, one contains the other, and it is sufficient to select the smaller one and discard the larger one. To demonstrate this property, suppose that we have two frontier nodes p and q of the same catchment basin, with strength $S(p) \leq S(q)$. If the intersection between their receptive regions is not empty, $SR(p) \cap SR(q) \neq \emptyset$, then there exists at least one node r common to them. For this to happen, the cost from r to p must be smaller than $S(p)$ and the cost from r to q must be smaller than $S(q)$. As $S(p) \leq S(q)$, then all the nodes in $SR(p)$ will have a smaller cost path to q than $S(q)$ and will belong to $SR(q)$, so $SR(p) \subseteq SR(q)$. In this case we call $SR(q)$ *redundant receptive region*. The complement of all redundant receptive regions are called *dead zones*.

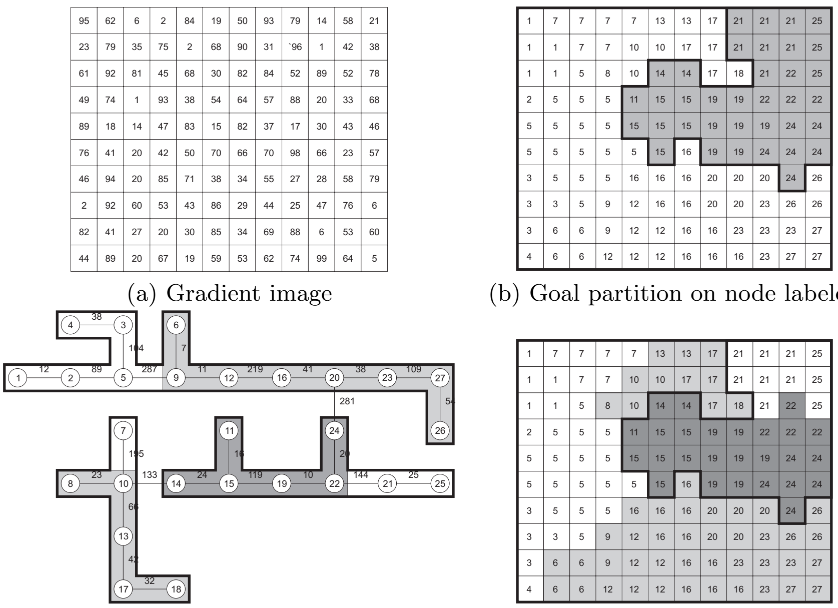
The solution of the minimal set of markers is to build the set of markers from one node of each non redundant receptive region of all catchment basins. If an additional marker of the same catchment basin label is placed in a dead zone of that catchment basin, the watershed result will not change.

3.1 Receptive Regions of the Multi-scale Watershed

In the case where the goal partition is one scale of the multi-scale watershed, there is an interesting result for the set of minimal markers. From the characterization of the multi-scale watershed at scale s, the strength of all frontier nodes will be greater than s and all the arc weights inside any catchment basin will be smaller than s. The receptive regions of all frontier nodes within the catchment basin will be all the same. This property has the practical consequence that for the multi-scale watershed segmentation, the minimum set of markers is made by picking any single node of each region. There is no dead zones in any scale of the multi-scale watershed transform.

3.2 Examples

To illustrate the computation of the receptive regions, a simple example is shown in Fig. 3. In this example we use the watershed using the volume as the flooding criteria. Fig. 3a shows the input image, normally a gradient image. There are 27 regions at the finest partition which are the number of nodes of the MST. Fig. 3b shows the labeled image where each label correspond to a node number on the graph. The goal partition is marked: two parts, the white and gray. Fig. 3c shows the MST. Nodes 10, 14, 20, and 24 are frontier nodes. There are three receptive regions which are indicated by shaded areas. Fig. 3d shows the receptive regions on the labeled image. The minimal set of markers are any three nodes, one from each receptive region.



(a) Gradient image

(b) Goal partition on node labeled

(c) MST and receptive regions

(d) receptive regions

Figure 3. Illustration of the computation of receptive regions.

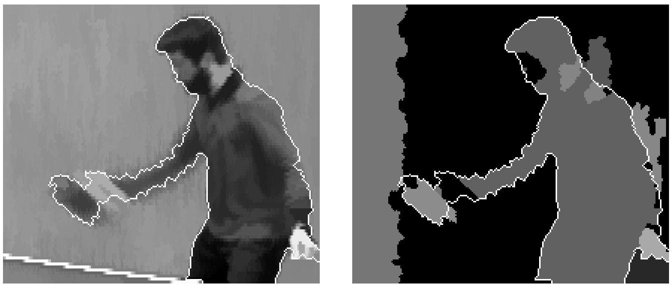
3.3 Further Restriction to Find Minimal Set of Markers

There are many sets of minimum markers that satisfy the solution described above. Depending on the application it may be desirable to impose additional constraints to find a particular set of markers. The constraints we found most interesting are: (i) a regional minimum node with larger extinguishing value; (ii) a regional minimum node most distant to the catchment basin boundary. The set of markers based on these constraints are in principle more robust to the segmentation. One can expect that if there is a small change on another image, as happen with image sequences, these markers would have more chance to reproduce the same segmentation.

4. Applications of the Minimal Set of Markers

We have identified two applications of the minimal set of markers: (i) aid in the interactive watershed; (ii) evaluation of pre-processing filters for segmentation.

Figure 4 shows a real example of a smoothed image where the goal is to delineate the figure of the tennis player and its racket, shown on 4a. The receptive regions required for this partition is shown on Fig. 4b. The dead zones are represented in black. In total, there are 12 receptive regions, depicted as different gray levels: 5 for the background and 7 for the object. This information is quite useful to understand where the most critical markers must be placed.



(a) Goal partition (b) Receptive regions

Figure 4. Receptive regions on a real image. Black depicts dead zones.

We designed an interactive watershed based segmentation that shows the receptive regions each time a new marker is placed on the image with the mouse. The visualization of the receptive regions is useful because to refine the segmentation until the goal partition, it is better to place a new marker in a receptive region of a different catchment basin than to place it in dead zone.

Depending on the smoothing filter used for pre-processing before the watershed transform, the number and size of receptive regions change, and by comparing their number, shape and position, one can evaluate the appropriateness of the filter used.

5. Conclusions

We review the watershed transform as a minimum spanning forest and study the inverse problem of the watershed from markers. Given a goal partition obtained with the watershed, find the minimal set of markers to reproduce the goal partition. We introduce the concept of receptive regions and dead zones associated to this problem. The receptive regions are the places which require at least one marker to obtain the goal partition and the dead zones are regions where the placement of a further marker of the same catchment basin label will not change the result of the watershed. We compute the receptive regions and dead zones using the minimum spanning tree representation of the watershed transform. An interesting property of the multi-scale watershed that we found is that it has no dead zones. There are mainly two application for the receptive regions: help in the interactive segmentation, and comparison of the performance of smoothing filters as pre-processing for image segmentation.

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