IMAGE SEGMENTATION BASED ON VISCOUS FLOODING SIMULATION

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Abstract The morphological approach to image segmentation is based on the watershed transform. It is a very powerful tool which presents many advantages: it is not parametric, computationally efficient... However, in comparison with energy-based methods as active contours, it does not allow to control the smoothness of the result. In order to introduce geometrical regularization constraints in the morphological segmentation scheme, two options are possible. The first one consists in simulating a viscous flooding for the construction of the watershed. The second one consists in computing the watershed on a smooth relief. In this article, the second alternative is chosen. The relief is modified so that its non viscous flooding is equivalent to the viscous flooding of the original relief. This choice allows to clearly separate the smooth procedure from the strict watershed computation and thereby to preserve the qualities and speed of the watershed transform. Performances of our regularization method are illustrated and discussed on an ultrasound imaging application.

Keywords: Watershed Transform, Viscous Flooding, Image Segmentation.

1. Introduction

Segmentation is one of the most fundamental problems in image analysis. Its goal is to extract the contours of one or several regions of interest in an image. Depending on the context, a region of interest will be characterized differently using for example its gray level, its contrast, its texture, its shape, its size... The segmentation method must allow the incorporation of each information needed in order to produce the desired result.

Among all the segmentation methods, the watershed transform is certainly one of the most popular judging by its capability to adapt itself to very different type of images [10, 11]. The image being seen as a topographic relief, the watershed transform involves the simulation of a relief flooding and the computation of watershed lines. The choice of the relief entirely determines the localization of the watershed lines, i.e the segmentation. The big issue is to build the "good" relief using the available a priori knowledge.

Contours generally correspond to crest lines of the gradient norm of the original image. When they are noisy or badly defined, the segmentation must result from a compromise between a complete adherence to the data (and possibly to the noise) and a certain amount of modelling. In a such situation, energy-based methods have an advantage because smoothness terms are incorporated in the model itself [5, 3, 14, 2, 16]. On the contrary, the morphological approach leaves the choice between two different options. The first one is closed to the energy-based approach. It consists in incorporating smoothness terms in the watershed model itself by simulating the behavior of a viscous fluid for computing the watershed [13, 4]. The second solution aims to modify the relief itself. This second option allows to clearly separate two steps: in one hand, the incorporation of the smoothing constraints into the topographic relief itself and in an second hand, the plain watershed. The benefits of the watershed transform (its computation efficiency, its non parametric aspect...) are preserved. In the present article, this second alternative is chosen.

This paper is organized as follows: in the following section, some key points of the watershed-based segmentation are briefly recalled and discussed. Then, the viscous closing of a relief is defined. In a last section, properties and performances of our segmentation method are presented and analyzed.

2. The watershed-based segmentation

2.1 Flooding simulation and watershed lines

Since its use for segmentation by Lantuejoul in 1979, several definitions of the watershed have been proposed by different authors [1, 10, 9, 12]. Very precisely, the watershed transform is a skeleton by influence zone associated to a certain distance: the topographic distance. From a pragmatic point of view, the watershed transform computation involves a flooding simulation and the computation of watershed points. This vision will be needed in this paper. Let us detail it.

The image is seen as a topographic relief: the function values are interpreted as altitudes, the regional minima are located at the bottom of the valleys. It is supposed that the contours to be extracted in the image correspond to crest lines of the relief (see figure 1). The topographic relief is progressively flooded by a number of lakes of increasing altitude. If each regional minimum defines a flooding source, different lakes will appear. All over the flooding process, each lake will take the exact shape of the valley. When two lakes of different sources meet, we are on a point of the watershed line. The flooding process is pursued till the relief is entirely flooded [7, 15](see figure 1).

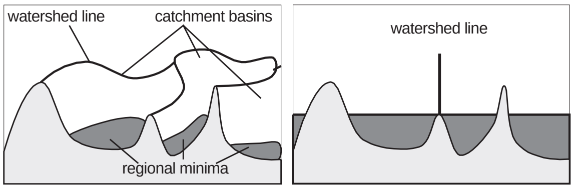

Figure 1. The topographic analogy: points where two lakes meet define the watershed line.

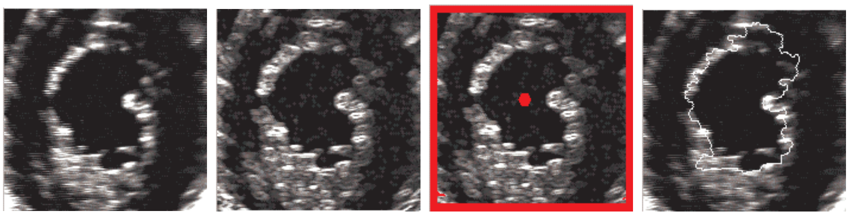

Figure 2. From left to right: original image, relief to be flooded (morphological gradient), flooding sources (in gray), watershed line

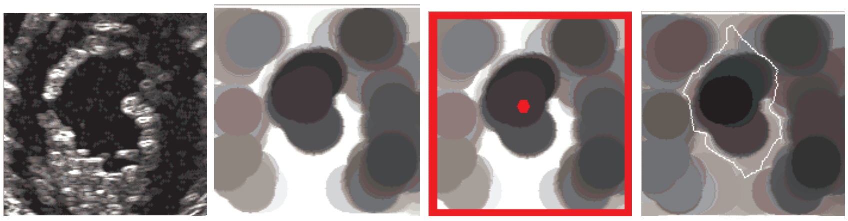

Figure 3. From left to right: original gradient, closed gradient (closing by a disk of radius $r_0 = 25$), flooding sources (in gray) and watershed line

2.2 The morphological closing: a classical pre-processing

The watershed transform allows to segment an image if the contours correspond to crest lines of the flooded relief and if flooding sources are placed inside the regions of interest. For example, let us consider the ultrasound image of the left ventricle presented in figure 2. Two flooding sources have been manually placed: one inside and one outside the ventricle. The relief to be flooded is the morphological gradient of the original image (dilation minus erosion). Note that in this case especially, the flooding of the original image were possible.

Without pre-processing, the watershed line is poorly localized. This was foreseeable: except for experts, the precise localization of the ventricle contour is quite impossible because of the noise and of the heart motion. During the flooding procedure, water leaks between fragment contours and some lakes may meet at wrong places. This phenomenon is quite frequent in the case of noisy data (and especially when gradient images are considered).

A classical solution to prevent water leak consists in closing the contours by computing a morphological closing. It is recalled that the morphological closing of a set A using a symmetrical structuring element B is defined by :

$$\varphi_B(A) = \bigcap_{x \in E} \{B_x \mid B_x \cap A \neq \emptyset\}$$

where B_x denotes the structuring element B centered at point x and where E denotes the image space. If B is a set, the closing of a function f may be easily defined by closing each section of f.

The result obtained in our example and presented in figure 3 illustrates a well known duality: without pre-filtering the segmentation is precise but noise sensitive ; with pre-filtering the segmentation is robust but less precise. Our purpose here is to attempt to gain simultaneously in precision and in robustness. The solution suggested here and presented in the next section is based on the construction of a more adequate modified relief inspired from the idea of a viscous flooding simulation.

3. Viscous flooding simulation

The aim of this part is to modify the original topographical relief in a such way that the level lines of the non viscous flooding of the modified relief are the same as the level lines of the viscous flooding of the original relief. A relief is decomposed into several elementary cylinders. The case of a single cylinder is firstly treated and then extended to the case of a complex function.

3.1 Case of a single cylinder

Considering the cylinder of complex geometry presented in figure 5, the object of this part is to simulate its flooding by a viscous fluid.

The concept of viscous flooding is due to F. Meyer [8]. It is inspired from a geophysical analogy originally proposed by G. Matheron for introducing the morphological opening [6].

In mathematical morphology, any opened set is assumed to be an union of disks of radius of increasing size. A smoothed version of a set may be obtained if a minimal size of the radius r is imposed: some details corresponding to disks of very small radius are not represented. This leads to the notion of opened set which is defined as the union of disks of radius r lodging in the original set:

$$\gamma_r(A) = \cup_{x \in E}\{B_x^r \mid B_x^r \subset A\}$$

γ_r is the opening by the structuring element B^r. If B^r is a disk, the opened set may be interpreted as the space filled by a viscous fluid at a given pressure [6]. Furthermore, openings by disks of decreasing radius may be used to represent the space filled by a viscous fluid at different pressure (see figure 4). One has only to decide how the radius will change with the pressure. Let us detail this phenomenon in the case of a cylinder of base S.

In the case of a non-viscous fluid (i.e. water), the cylinder is entirely flooded. In the case of a viscous fluid (i.e. mercury), certain point of the cylinder may

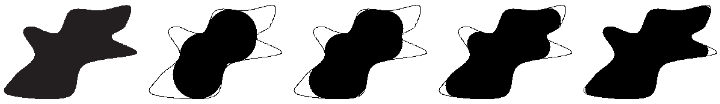

Figure 4. Available space and space filled by a more and more compressed viscous fluid (represented via openings by disks of decreasing radius)

not be reached especially if its base is of complex geometry (see figure 5). If h denotes the flooding level, at altitude h, the space filled by the viscous fluid is equal to the opened set $\gamma_{r_0}(S)$, where r_0 is the radius associated to the fluid at the atmospheric pressure. At an intermediate altitude k, the fluid being more compressed (because of the weight of the fluid column of high $(h-k)$), the fluid acts as if it were less viscous: the associated disk radius get smaller ; the opened set get closer to the original set: it is $\gamma_{r_{(h-k)}}(S)$ with $r_{(h-k)} \leq r_0$. At the limit, the fluid is non viscous ($r = 0$) and all the available space is filled ($r_t \to 0$ as $t \to +\infty$). From above, the geometry of the viscous lake results from the superposition of all intermediate sections of the viscous fluid column. It is:

$$\bigcup_{0 \leq k \leq h} \gamma_{r_{(h-k)}}(S)$$

We arrive now at the heart of the method: the construction of a container having the same level sets as the viscous lakes formed in our cylinder. Noting that our viscous fluid column forms a kind of stalagmite, the equivalent container is simply obtained by inverting our stalagmite (see figure 5):

$$\bigwedge_{0 \leq k \leq h} \varphi_{r_k}(f+k)$$

f denotes the lowest cylinder. The closing of highest activity ($r(t) = r_0$) is localized at the lowest level ($k = 0$). On the contrary, the closing of lowest activity ($r(t) \to 0$) is localized at the highest level (it acts on a translated cylinder $f + k$).

This relation is valid for all flooding levels h, so the equivalent container is:

$$T(f) = \bigwedge_{k \geq 0} \varphi_{r_k}(f+k)$$

3.2 Case of a complex function

The case of a more complex function is simply deduced from the precedent case. The function is decomposed into a family of elementary cylinders (the basis of the cylinder corresponds to the connected components of the level sets of the function) (see figure 6):

$$g = \bigwedge_{i \in I} f_i$$

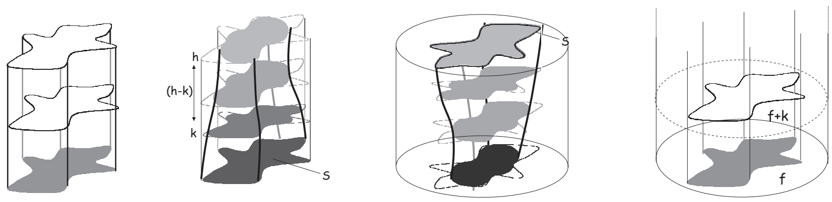

Figure 5. From left to right: cylinder to be flooded, column of viscous fluid, container having the same level sets as the viscous lakes, the level sets are the bases of translated cylinders

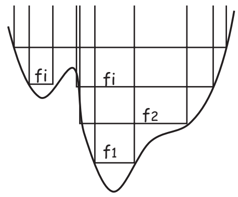

Figure 6. Decomposition of a relief into several cylinders

where g denotes the function and $\{f_i\}_{i \in I}$ the family of cylinders. The basis of the cylinder f_i is localized at the level i. Each cylinder is flooded individually creating a stalagmite in each. The final equivalent relief is obtained by inverting each stalagmite and by considering the lowest stalactites:

$$T(g) = \bigwedge_i T(f_i)$$

resulting in:

$$T(g) = \bigwedge_i \left(\bigwedge_k \varphi_{r(k)}(f_i + k) \right) = \bigwedge_k \left(\bigwedge_i \varphi_{r(k)}(f_i + k) \right)$$

and finally:

$$T(g) = \bigwedge_k \varphi_{r(k)}(g + k)$$

T is finer than the standard morphological closing: $f \leq T(f) \leq \varphi_{r_0}(f)$. It will be called a viscous closing. Note that the image is first roughly closed ($\varphi_{r_0}(f)$), then details are re-injected via softer closings ($\varphi_{r_t}(f+t)$ with $r(t) \leq r(0)$). As the flooding level h increases ($h \rightarrow \infty$), the fluid at the basement is more and more compressed (it seems like a non-viscous fluid ($r_h \rightarrow 0$), the data are less and less filtered ($\varphi_{r_h}(f+t) \rightarrow f+h$).

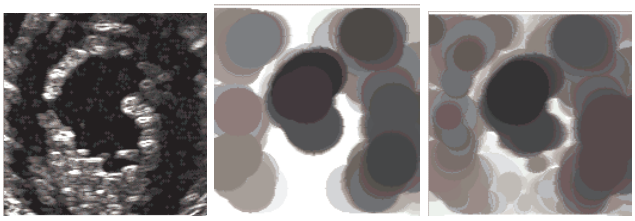

Figure 7. From left to right: original relief, relief closed by a morphological closing (of size $r_0 = 25$), relief closed by a viscous closing ($T(f) = \wedge_{t \geq 0} \varphi_{r(t)}(f - t)$ with $r(t) = r_0 e^{-\lambda k}$, $r_0 = 25$, $\lambda = 0.1$)

4. Application to image segmentation

4.1 Case of a unique flooding source

In the case of a unique flooding source, the strategy is very closed to the strategy developed in active contours theory: an "active variety" evolves inside or outside an object till its position coincides with the contour of the object. In our case, the active variety is a viscous fluid slick. A flooding source is placed inside the ventricle. The simulation of the viscous flooding involves:

1 the computation of the viscous closing : $f_v = \bigwedge_{t \geq 0} \varphi_{r(t)}(f + t)$

2 the threshold of the closed relief at level h: $S_h = \{x \in E,\, f_v(x) < h\}$

3 the extraction of the border of the connected subset of S_h containing the marker of the ventricle.

Two parameters have to be chosen: the flooding level h and the values of the radius $r(t)$ when t varies between 0 and h. Note that this parameters are similar to those classically imposed in active contour models.

The choice of the function $r(t)$ may not be linked to any data or a priori knowledge. In this paper: $r(t) = r_0 e^{-\lambda k}$, where r_0 defines the viscosity under the atmospheric pressure ($r_0 = 25$) and where λ is a constant (here $\lambda = 0.1$ what guaranties $r(h) \rightarrow 0$ when $h \rightarrow +\infty$ at the scale of the data). This function has been chosen because it is simple but others functions may be imagined. Note that the viscous closing is not invariant by contrast change as it was the case for the morphological closing (r derives from the levels t). This explains the importance of the scale parameter λ.

The last parameter to be fixed is the final level h. In active contour models, it is fixed by searching a local minimum of the evolution function. In our example, the final level is fixed in the following way: the watershed is firstly computed on the closed gradient (by a disk of radius r_0). It is assumed that the result is closed to the desired contour. h is then fixed as the average of the gray levels of all watershed points.

The segmentation obtained by our method is presented and compared to the segmentation obtained by a geodesic active contour [2] (see figure 8). The results are similar. This were foreseeable: the evolution of a viscous lake may

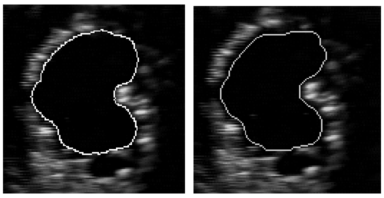

Figure 8. From left to right: segmentations based on a viscous flooding and segmentation based on a geodesic active contour

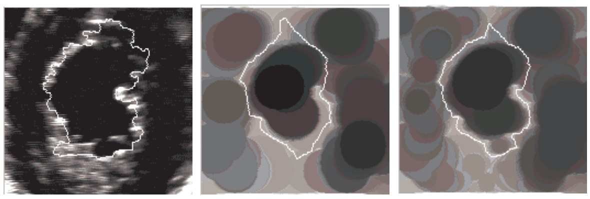

Figure 9. From left to right: watershed line computed on the original relief, on a closed relief and on a relief modified by viscous closing

be interpreted as an active variety, the viscosity playing the role of the rigidity. The difference between the two approaches lies on the mathematical formulation: the geometrical constraints have been expressed as pre-processing. active contour,geodesic

4.2 The viscous watershed

If several flooding sources are considered, it is possible to define a kind of viscous watershed by computing the watershed associated to the relief modified by viscous closing. The markers being the same as in figure 2, the segmentation obtained is presented in figure 9 and compared to the segmentation obtained by standard methods. The result is not so smoothed than expected. The reason is the following: two viscous lakes may meet only if they are strongly compressed. But in that case, the result is not smoothed anymore. Several solutions may be imagined: for example, the flooding process may be stopped before the relief is entirely submerged.

5. Conclusion

This article presents a new segmentation method based on a viscous flooding simulation. The viscosity is introduced by the way of a family of openings of decreasing radius. It is proved that the viscous flooding of a certain relief equals to a standard flooding of a filtered relief: the filtering procedure is called a viscous closing. From a segmentation point of view, the viscosity plays the same role as the rigidity in the active contours: the results obtained by these methods are very similar. Moreover, the treatments being not symmetrical, the definition of a kind of viscous watershed transform is not easy. This question has yet to be solved.

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