# ON EDGE DETECTION, EDGE INTEGRATION AND GEOMETRIC ACTIVE CONTOURS

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**Abstract** We recently realized that the Marr-Hildreth edges, computed as the zero crossings of the image Laplacian, can be viewed as optimal edge integration curves solving a geometric variational problem. We used this observation to derive a new set of edge integration and object segmentation procedures. Here we show that the edge detectors proposed by Haralick, and subsequently claimed to be optimal, in some sense (based on 1D criteria) by Canny, and then numerically and computationally enhanced by Deriche, can also be interpreted as optimal edge contours whose normals align with the image gradient field, and further satisfy a topological uniformity measure inside a closed region defined by the contour. The combination of these two measures yield a robust edge detection/integration procedure, as well as better geometric curve evolution based segmentation procedures. In other words we provide 2D variational reasoning for the classical Haralick/Canny/Deriche-like edge detectors. We show how to use this new formulation for introducing novel geometric segmentation by curve evolution.

**Keywords:** Laplacian zero crossing, Canny edge detector, curve evolution, PDEs, variational calculus.

## 1. Introduction

The second derivative along the gradient direction and the second derivative along an image level set sum up to the rotationally invariant laplacian operator. The first to recognize the importance of processing images by treating differently these two components was Gabor in the $60^{th}$, in a paper that appeared too early to be fully understood and appreciated [8, 13] (see also [12] for an extension of Gabor's inverse diffusion across the edge for the color case.)

Marr and Hildreth considered the zero crossings of the laplacian of the image as an edge detector [16, 15]. As it was always considered wise to smooth noise by a convolution with a gaussian kernel, the edges of Marr and Hildreth were defined as the zero crossing curves of the laplacian of gaussian (LOG) operator applied to the image. The gaussian convolution kernel has some nice


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properties. In 1D, for example it has the narrowest spectral times spatial bandwidth. However, convolution with a two dimensional gaussian can disconnect connected components, and thereby increase the topological complexity of the objects appearing in a gray level image. This property motivated the invention of geometric scale space and geometric image processing [1, 19, 20]. These novel geometric filters were defined to preserve the topology of the gray level sets of an image, while selectively smoothing the image.

Haralick [9], was the first to recognize the importance of the zero crossing of the second order derivative along the image gradient direction as an edge detector. Next, Canny [2] attempted to motivate Haralick edge detector with a variational principle that holds in 1D, but the extension to two dimensions remained heuristic. Canny made the important observation that using only the gradient direction component of the laplacian can decrease the influence of additive random noise. He argued that it happens due to the elimination of the second order derivative along the level sets that does not contribute to its localization. This argument holds only if a good estimator for the gradient direction is provided, which is again an operation sensitive to noise. Hence, Canny's one dimensional localization analysis applies only in one dimension and actually has very little to do with the experimental success of this operator in image analysis. Following Canny, Deriche [6] introduced an efficient numerical approach for the pre-filtering step proposed by Canny. We will show here that the Haralick edge detector does however have an interesting variational meaning in two dimensions.

## 2. Haralick-Canny-Deriche-Like Edge Detectors

As in Marr-Hildreth edge detectors some smoothing of the image is performed first; then the Haralick edge detector [9, 2, 6] is defined as the zero crossings of the second derivative along the image gradient direction. Given the gray level image $I(x, y) : I\!R^2 \rightarrow [0, 1]$, define the gradient direction vector field

$$\vec{\xi}(x, y) = \frac{\nabla I}{|\nabla I|} = \frac{\{I_x, I_y\}}{|\nabla I|},$$

and the orthogonal vector field,

$$\vec{\eta}(x, y) = \frac{\bar{\nabla} I}{|\nabla I|} = \frac{\{-I_y, I_x\}}{|\nabla I|},$$

so that $\langle \vec{\eta}, \vec{\xi} \rangle = 0$. See Figure 1.

The Haralick edge detector finds the image locations where both $|\nabla I|$ is larger than some threshold and $I_{\xi\xi} = 0$. This procedure was observed to yield better results compared to the zero crossings of the laplacian. We will not review the justifications given before, yet, to the best of our knowledge there exist no rigorous explanations or analysis for this operator in two dimensions, and the improved results obtained with the Haralick edge detector were explained, as we stated before, based on some one dimensional arguments.

Let us try to justify the good properties of the HCD-edges from a variational point of view. We formulate the edge operator as a result of a variational

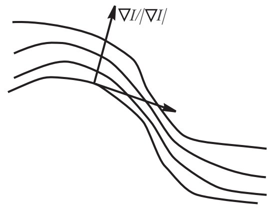


*Figure 1.* The level sets of $I$, the gradient, $\vec{\xi}$, and the orthogonal, $\vec{\eta}$, directions.

principle in 2D and in the process we shall get a better understanding and a rigorous meaning for the Haralick edge tracing operator. First, we use the rotation invariance property of the laplacian to define the operator

$$I_{\xi\xi} = I_{\xi\xi} + I_{\eta\eta} - I_{\eta\eta} = \Delta I - I_{\eta\eta}.$$

The edge detector provides curves along which $\Delta I - I_{\eta\eta} = 0$. We have shown in [11] that $\Delta I \vec{n} = 0$ is the result of maximizing the geometric integral measure

$$\oint_0^L \langle \nabla I, \vec{n} \rangle ds,$$

where $s$ is the arc length parameter of the curve, $\vec{n}$ its normal, and $L$ its total length. Let us define, as usual, $\vec{n}, \kappa$, and $\vec{t}$ to be the unit normal, the curvature, and the tangent of the curve $C$, respectively. We have that $\kappa\vec{n} = C_{ss}$, and $\vec{t} = C_s = C_p/|C_p|$. Let $\Omega$ be the domain inside the curve $C$, see Figure 2.

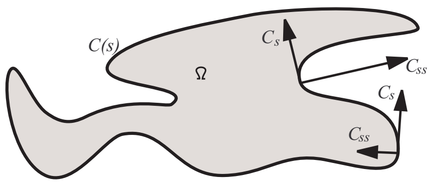


*Figure 2.* A closed curve $C$, with $C_s$ the unit tangent, $C_{ss}$ the curvature vector, and $\Omega$ the area inside the curve.

The quantity $I_{\eta\eta}$ is known to be the curvature of the image level set multiplied by the image gradient magnitude. That is,

$$I_{\eta\eta} \quad = \quad |\nabla I| \mathrm{div} \left( \frac{\nabla I}{|\nabla I|} \right) = \kappa_I |\nabla I|,$$

where $\kappa_I$ is the curvature of the image level sets. In order to understand the meaning of the Haralick operator, we search for a geometric functional along a curve that yields $I_{\xi\xi}\vec{n} = 0$ as an Euler Lagrange equation. For that, we may equivalently search for the geometric functional along a curve that yields

$(\Delta I - I_{\eta\eta})\vec{n} = 0$ as an Euler Lagrange equation. Consider the general scalar cost function

$$\iint_\Omega g(x,y)dx\,dy,$$

that integrates the cost $g$ inside the curve $C$. Then, by Green's theorem, the EL equation is given by $g(C)\vec{n} = 0$, see [21, 18]. If we set, $g(x,y) = I_{\eta\eta}$, we have the required result.

Let us look into the geometric meaning the second part in the Haralick operator

$$\iint_\Omega I_{\eta\eta}dx\,dy \quad = \quad \iint_\Omega \kappa_I|\nabla I|dx\,dy = \int_{I\!R}\left(\int_{I^{-1}(u)\cap\Omega}\kappa_I ds\right)du,$$

where we used the co-area equation [7] to change coordinates from $dx\,dy$ to $ds\,du$, where $u = I(x,y)$. Here $s$ is the arclength of the image level sets, and $u$ represents its gray values. For a closed level set contour we have that $\oint \kappa_I ds = 2\pi$. Therefore, the integral over $I_{\eta\eta}$ inside $C$, measures the topological uniformity of $I$ in $\Omega$. Figure 3 shows the result of integration over $I_{\eta\eta}$ in a closed contour for various image surfaces. We see that this is a measure generalizing the 1D total variation and is a function of the topological–complexity of $I$ over $\Omega$.

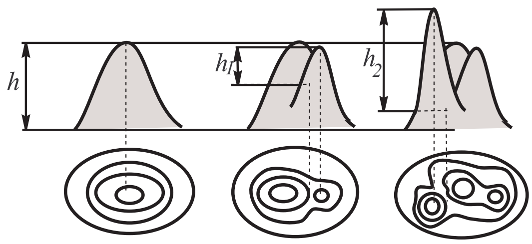


*Figure 3.* Integration of $I_{\eta\eta}$ in closed contours for various image surfaces. The result is, left: $2\pi h$, middle: $2\pi(h+h_1)$, and right: $2\pi(h+h_1+h_2)$.

Putting it all into a unified framework, we have that the Haralick operator searches for alignment of the edge normals with the image gradients, while also minimizing a topological uniformity and total variation like measure inside the contour. The only difference between the Marr-Hildreth laplacian zero crossings [16, 15] and the Haralick operator [9, 2, 6], is the additional above minimization term of topological complexity that is required inside the detected edge contour. This extra term prefers to group together uniform areas, avoids noisy regions, and thereby has the chance to yield more robust edge curves or better object segmentation.

The uniformity measure expressed by $\iint I_{\eta\eta}dx\,dy$ is just one possible choice that may be appropriate to some edge detection and integration problems. We could add to our geometric measure other useful terms. Probably the simplest example could be based on some knowledge of the gray levels of objects expected to appear in a scene. If we know that the objects we try to segment are expected to have a constant gray level $c_1$, and the background is expected to be $c_2$, then the functional we would like to add to the cost function could be defined by

$$\begin{aligned}\int_{\partial\Omega_1} \langle \nabla I, \vec{n}\rangle ds \quad &- \quad \iint_{\Omega_1} \kappa_I |\nabla I| dx\, dy \\ &- \quad \alpha \left( \iint_{\Omega_1} (I - c_1)^2 dx\, dy + \iint_{\Omega\setminus\Omega_1} (I - c_2)^2 dx\, dy \right).\end{aligned}$$

As Euler-Lagrange equations we then get the edge detector operator

$$\left( \Delta I - I_{\eta\eta} + \alpha \left( I - \frac{c_1 + c_2}{2} \right) \right) \vec{n} = 0,$$

which is a trade-off between gradient alignment coupled with topological uniformity, and the simplest segmentation we can think of in gray level: thresholding!

## 3. Active Contours

Motivated by classical 'snakes' [10], non-variational geometric active contours [14, 3], and the 'geodesic active contours' [4], we follow [11] and use the Haralick operator as part of a new geometric active contour model. We search for simple parametric planar curves that map their arc length interval $[0, L]$ to the image plane, such that $C : [0, L] \to I\!R^2$, or in an explicit parametric form $C(s) = \{x(s), y(s)\}$. Here again $s$ is the arc length parameter, and we have the relation between the arc length $s$ and a general arbitrary parameterization $p$, given by

$$ds = \sqrt{\left(\frac{dx(p)}{dp}\right)^2 + \left(\frac{dy(p)}{dp}\right)^2} dp = |C_p| dp.$$

Consider the geometric functional

$$\psi(C) = \oint_0^L \left\langle \vec{V}, \vec{n} \right\rangle ds - \iint_\Omega I_{\eta\eta} dx\, dy.$$

The first term is an integration along the curve $C$ that measures the alignment of a vector field $\vec{V} = \{u(x, y), v(x, y)\}$ with the curve normal $\vec{n}$, where for example we can take $\vec{V} = \nabla I(x, y) = \{I_x, I_y\}$ as the gray level image gradient. As we have seen, the second term measures the topological uniformity inside the curve. Our goal is to find curves $C$ that maximizes the above geometric functional.

In a general parametric form, we have the following re-parameterization invariant measure

$$\psi(C) = \oint_0^1 \left\langle \vec{V}, \vec{n} \right\rangle |C_p| dp - \iint_\Omega I_{\eta\eta} dx\, dy.$$

The Euler Lagrange (EL) equations $\delta\psi(C)/\delta C = 0$ should hold along the extrema curves. Where for $\vec{V} = \nabla I$ we have $\delta\psi/\delta C = (\Delta I - I_{\eta\eta})\vec{n}$. We could easily add terms to our functional, like the geodesic active contour model, that could now play the role of regularization.

In order to determine conditions for optimal curves in the plane, we need to solve the EL equations. In the next section we shall follow the geodesic active contour philosophy, see [4], and design a curve evolution rule that is given by

$$C_t = \frac{\delta\psi(C)}{\delta C}.$$

This is a gradient descent rule with respect to the chosen cost functional.

## 4. Gradient Descent via Level Set Formulation

We embed a closed curve in a higher dimensional $\phi(x, y)$ function, which implicitly represents the curve $C$ as a zero set, i.e., $C = \{\{x, y\} : \phi(x, y) = 0\}$. This way, the well known Osher-Sethian [17] level-set method can be employed to implement the curve propagation toward their optimal locations.

Given the curve evolution equation $C_t = \gamma\vec{n}$, its implicit level set evolution equation reads

$$\phi_t = \gamma|\nabla\phi|.$$

The equivalence of these two evolutions can be easily verified using the chain rule and the relation $\vec{n} = \nabla\phi/|\nabla\phi|$,

$$\phi_t \quad = \quad \langle\nabla\phi, C_t\rangle = \langle\nabla\phi, \gamma\vec{n}\rangle = \gamma\left\langle\nabla\phi, \frac{\nabla\phi}{|\nabla\phi|}\right\rangle = \gamma|\nabla\phi|.$$

Thereby, the explicit curve evolution as a gradient descent flow for $\vec{V} = \nabla I$ is given by

$$C_t = (\Delta I - I_{\eta\eta})\,\vec{n},$$

for which the implicit level set evolution is given by

$$\phi_t = (\Delta I - I_{\eta\eta})\,|\nabla\phi|.$$

## 5. Simulation Results

Figure 4 compares the more robust edge detector given by the zero crossings of $I_{\xi\xi}$, and the zero crossings of the laplacian. It is obvious that $I_{\xi\xi}$ is less sensitive to the noise.

Next, we compared between edge integration results of the Haralick-like and the laplacian terms within the geometric active contour model. Comparing the results of the two methods clearly demonstrates the regularization effect of the Haralick-like term. While the laplacian term causes the contour to oscillate and capture insignificant small structures, the Haralick-like term regularizes the propagating curve and leads to smooth boundary curves. In all cases we started from the image frame as the initial contour, and applied a multi-resolution coarse to fine procedure, as in [18], to speed up the segmentation process.

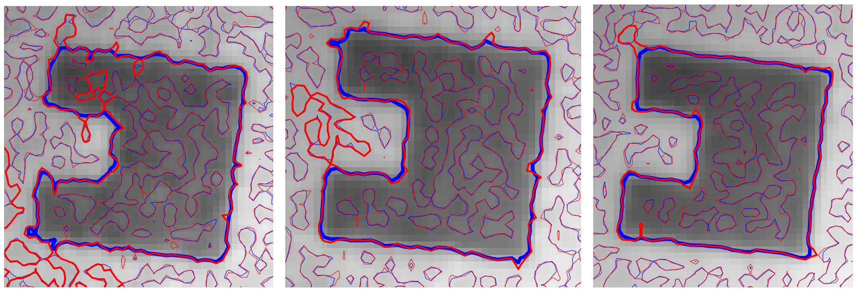

*Figure 4.* Zero crossings of $I_{\xi\xi}$, in blue, compared to zero crossings of $\Delta I$, in red, for edge detection of an object with various noise levels.

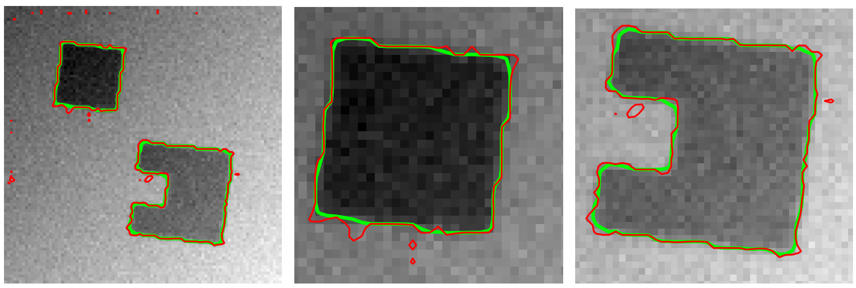

*Figure 5.* Synthetic images with a tilted intensity plane added to the original image and additive gaussian noise. The edge integration via geometric active contour results for both the laplacian term (in red) and the second order derivative along the gradient direction Haralick-like term (in green) are shown in two resolutions.

## 6. Conclusions

We derived the Haralick operator (also known as the Canny-Deriche edge detector) from a two dimensional geometric variational principle. Unlike the one dimensional analysis of Canny, the proposed analysis explains the stability of the Haralick-like edge detector and reveals rigorously the fundamental reasons for its regularization effect.

The proposed variational principle explains the robustness of the Haralick operator compared to the Marr-Hildreth edge detector. Most importantly, it enables us to design new and better edge detectors and active contours for images. It shows the way to use similar variational principles to design new edge detectors in which homogeneity can be defined differently. Specifically, we showed how to use the Haralick-like term as part of a geometric active contour model, that improves the performances of the classical geodesic active contour in cases where the homogeneity as defined by the variational principle is significant. Our analysis also shows the direct link between edge detection and edge integration processes that incorporate uniformity as part of their measure, a direction that was actually started by the work of Chan and Vese [5].

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