

# How much would a blending stockpile reduce variation?

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## Abstract

A blending stockpile generally reduces the amount of grade variation between parcels of material of a moderate size, because the parcels of material on reclaiming are mixtures of the parcels of material which were stacked. This general principle will be illustrated using both mathematical and computer models for the geometry of a stockpile and using both actual grade data and a variogram to describe the input variation. Some applications to the planning of mining operations will be briefly discussed.

*Key words:* Blending, sampling, planning of mining operations

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## 1 Introduction

In materials handling operations, stockpiles often serve three purposes. First, stockpiles act a buffers, so that processes before them and processes after them can operate without being constrained by each other. This function often provides the primary economic justification for the expense of stockpiling, so blending options must be considered by looking at minor modifications to the stockpiles which have been planned in order to provide buffering. Models for investigating the buffering function are generally models which consider reliability and scheduling.

Second, stockpiles can be used to reduce the amount of variation in grade within stockpiles. This means that the variation in the grade of the material over a time or tonnage scale that matters should be smaller in the flow of

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material which is reclaimed from the stockpile than it was in the flow of material which was stacked onto the stockpile. This is the blending function which is the topic of most of this paper.

Third, stockpiles can be used as part of a grade targeting system to reduce the amount of variation in grade between stockpiles. Such a system relies on being able to measure the grade of material as it is stacked and having some ability to procure material which is likely to adjust the average grade in a desired way. This is a current research interest.

Many types of models for making useful predictions about the amount of variation in reclaimed material will be discussed in this paper.

- (1) The simplest model of blending is that material will be perfectly mixed. This model may be adequate for some purposes, but it does not provide any useful prediction of the amount of variation which is likely to occur in the reclaimed material.
- (2) Sampling theory, as discussed in Gy (1982), tells us about the minimum amount of variation which could possibly be achieved if all the material in a stockpile were mixed. A self-contained derivation of one particularly relevant formula is given in section 2 of this paper.
- (3) Bed-blending theory, as discussed in Gy (1981), tells us about the variation which would occur if the geometry of stacking and reclaiming were perfect in the sense that each parcel of stacked material was equally represented in each parcel of reclaimed material. A self-contained derivation of the expected output variation is given in section 3.
- (4) Blending based on realistic geometries is discussed in sections 4 to 7. A novel part of this is the direct calculation of variances of output grades in section 4.1.

Sections 8 and 9 of the paper discuss contexts in which the blending performance of stockpiles is likely to be considered and briefly describe some applications.

## **2 Sampling theory: The best possible blending**

The concepts of Fundamental Error and Heterogeneity Invariant presented by Gy (1992) provided a bound on the best possible blending that can be achieved without reducing the sizes of constituent particles. This best possible blending would be achieved if all particles in a lot of material being blended had the same probability of being included in an output block. This is analogous to sampling where the best possible sampling is to give each particle the same probability of being included in the sample.

The following is a self-contained derivation of the minimum amount of variation, based on that given on pages 35–36 of Robinson (2000).

Suppose that a lot of material contains particles indexed by  $i$ , with particle  $i$  having mass  $m_i$  and grade  $g_i$ . The lot has total mass  $M = \sum_i m_i$  and average grade  $G = \sum_i m_i g_i / M$ .

In order to estimate the variance of the grades of samples in which particles are each selected with probability  $p$ , we look first at the variation between samples excluding particle  $i$  and samples including particle  $i$ . The particles other than particle  $i$  have total mass  $M - m_i$  and average grade

$$\frac{GM - m_i g_i}{M - m_i} = G + \frac{m_i(G - g_i)}{M - m_i} \approx G + \frac{m_i}{M}(G - g_i).$$

The approximation works provided that the mass of the single particle,  $m_i$ , is very small compared to the mass of the lot,  $M$ . This is the average grade of samples excluding particle  $i$ , which happens with probability  $1 - p$ .

With probability  $p$ , the sample will include particle  $i$ . It will also include an average mass of  $p(M - m_i)$  of other particles. On average, the sample will have mass  $p(M - m_i) + m_i$  and grade

$$\begin{aligned} \frac{p(GM - m_i g_i) + m_i g_i}{p(M - m_i) + m_i} &= G + \frac{m_i(1 - p)(g_i - G)}{pM - pm_i + m_i} \\ &\approx G + \frac{(1 - p)m_i}{pM}(g_i - G). \end{aligned}$$

The weighted mean grade is approximately  $G$  and the average squared departure of the grade from  $G$  is approximately

$$(1 - p) \left[ \frac{m_i}{M}(G - g_i) \right]^2 + p \left[ \frac{(1 - p)m_i}{pM}(g_i - G) \right]^2 = \frac{(1 - p)}{p} \left( \frac{m_i}{M} \right)^2 (g_i - G)^2.$$

The total sampling error variance,  $V_S$ , is the sum of such average squared departures for all individual particles, namely

$$V_S = \frac{(1 - p)}{p} \sum_i \left[ \left( \frac{m_i}{M} \right)^2 (g_i - G)^2 \right].$$

The average mass of the sample is  $pM$  since all particles have probability  $p$  of being in the sample. Denoting this by  $M_S$ , the sampling variance may be written as

$$V_S = \frac{(1 - p)}{M_S} \sum_i \left[ \frac{m_i^2}{M} (g_i - G)^2 \right].$$

Note that this is similar to the conclusion about this component of variation given by Gy (1981), although there is an extra factor of 3 in the denominator of a formula given on page 220 of Gy (1981). The fundamental error for the mixture will generally be greater than for the components of the mixture. The increase is substantial in cases when different, reasonably-uniform materials are blended.

This component of variation can never be eliminated. When predicting the amount of variation expected after a blending stockpile, this component of variation should be added to components of variation predicted by other means, because the other models do not take it into account.

### 3 Bed-blending

“Bed-blending” is a term used to describe systems whereby each reclaim slice includes as similar as possible an amount of material from all layers of material stacked. A mathematical model for bed-blending can be derived by considering a situation in which material is placed into a two-dimensional array of containers. We will ignore variation in the material flow rate and assume that the the same mass of material is placed in each container. Let us denote the average grades of the portions of material placed in the containers by  $X_{i,j}$ , where  $i$  is an index which runs through the  $s$  stacked layers in the bed-blending operation and  $j$  is an index which runs through the  $r$  batches which will be distinguished as the material is reclaimed. In terms of the array of symbols denoting average grades below, stacking consists of filling the containers row-by-row and reclaiming consists of emptying the containers column-by-column, mixing the material from all the containers in a column.

The usual convention for writing matrices is to put the first row on the top. Here the first row is put on the bottom, since the first material stacked on a stockpile always goes on the bottom and subsequent material is placed on top of it. The conventions used in this paper are that stacking always goes from bottom to top and reclaiming always goes from left to right.

$$\begin{array}{cccc}
 X_{s,1} & X_{s,2} & \dots & X_{s,r} \\
 \vdots & \vdots & & \vdots \\
 X_{2,1} & X_{2,2} & \dots & X_{2,r} \\
 X_{1,1} & X_{1,2} & \dots & X_{1,r}
 \end{array}$$

We will now derive a formula for the variation of the output grade which can



$$\begin{aligned}
& + V\left(\frac{h+(s-1)}{s}\right) + V\left(\frac{h+(s-2)}{s}\right) + \dots + V\left(\frac{h}{s}\right) \\
& - V\left(\frac{1}{s}\right) - V\left(\frac{2}{s}\right) - \dots - V\left(\frac{s-1}{s}\right) \\
& \quad - V\left(\frac{1}{s}\right) - \dots - V\left(\frac{s-2}{s}\right) \\
& \quad \quad \quad \ddots \\
& \quad \quad \quad \quad - V\left(\frac{1}{s}\right) \\
& - V\left(\frac{1}{s}\right) - V\left(\frac{2}{s}\right) - \dots - V\left(\frac{s-1}{s}\right) \\
& \quad - V\left(\frac{1}{s}\right) - \dots - V\left(\frac{s-2}{s}\right) \\
& \quad \quad \quad \ddots \\
& \quad \quad \quad \quad - V\left(\frac{1}{s}\right)
\end{aligned}$$

Hence

$$\begin{aligned}
s^2 V_O(h) &= sV\left(\frac{h}{s}\right) \\
& + (s-1)[V\left(\frac{h+1}{s}\right) + V\left(\frac{h-1}{s}\right) - 2V\left(\frac{1}{s}\right)] \\
& + (s-2)[V\left(\frac{h+2}{s}\right) + V\left(\frac{h-2}{s}\right) - 2V\left(\frac{2}{s}\right)] \\
& \quad \quad \quad \vdots \\
& + (1)[V\left(\frac{h+(s-1)}{s}\right) + V\left(\frac{h-(s-1)}{s}\right) - 2V\left(\frac{s-1}{s}\right)]
\end{aligned}$$

Now variograms always have the property that  $V(h) = V(-h)$  and we are only interested in output lags in the range  $0 \leq h \leq 1$ , so we can write this formula in a form such that the input variogram is evaluated only for positive lags.

$$\begin{aligned}
s^2 V_O(h) &= sV\left(\frac{h}{s}\right) \\
& + (s-1)[V\left(\frac{1+h}{s}\right) + V\left(\frac{1-h}{s}\right) - 2V\left(\frac{1}{s}\right)] \\
& + (s-2)[V\left(\frac{2+h}{s}\right) + V\left(\frac{2-h}{s}\right) - 2V\left(\frac{2}{s}\right)] \\
& \quad \quad \quad \vdots \\
& + (1)[V\left(\frac{(s-1)+h}{s}\right) + V\left(\frac{(s-1)-h}{s}\right) - 2V\left(\frac{s-1}{s}\right)] \tag{1}
\end{aligned}$$

It should be noted that equation (1) does not depend on  $r$ , except through the lag  $h$  which is expressed as a fraction of a stockpile. As  $r$  increases,  $h$  can take more finely-spaced values.

Some special cases of formula (1) are as follows.

- (1) For a linear input variogram, say  $V(h) = B|h|$ , the formula gives  $V_O(h) = s^{-2}V(h)$ . This result is not perfectly consistent with the conclusions of Gy (1981) but has essentially the same practical consequences. It can be checked by considering an input process which has constant grade except

for one jump of size  $\delta$  which occurs at a point which has a uniform distribution between the two ends of the stockpile-ful of material. For such a process the input variogram is  $V(h) = \frac{1}{2}\delta^2 h$ . The output process will have constant grade except for one jump of size  $\delta/s$  which occurs at a point which also has a uniform distribution between the two ends of the stockpile-ful. Hence the output variogram for lags between 0 and 1 is  $V_O(h) = \frac{1}{2}\delta^2 s^{-2} h$ .

- (2) For a periodic input process with input variogram  $V(h) = A[1 - \cos(Bh)]$ , the output variogram is as follows. It has been simplified using the fact that  $\cos(\alpha + \beta) + \cos(\alpha - \beta) - 2\cos(\alpha) = 2\cos(\alpha)[\cos(\beta) - 1]$ .

$$A^{-1}s^2V_O(h) = \left[1 - \cos\left(\frac{Bh}{s}\right)\right] \times \\ \left[s + 2(s-1)\cos\left(\frac{B}{s}\right) + 2(s-2)\cos\left(\frac{2B}{s}\right) + \dots + 2(1)\cos\left(\frac{(s-1)B}{s}\right)\right]$$

As could be expected by thinking about the process itself, the output variogram is also the variogram of a periodic process. The period of the output process is  $s$  times as large as that of the input process. The amplitude of the output process depends on the size of the expression  $s + 2(s-1)\cos\left(\frac{B}{s}\right) + 2(s-2)\cos\left(\frac{2B}{s}\right) + \dots + 2(1)\cos\left(\frac{(s-1)B}{s}\right)$ . At one extreme, if the period of the stacking process is a multiple of the period of the input process then  $\cos\left(\frac{B}{s}\right) = 1$ , the expression takes the value  $s^2$  and the output process has the same variogram as the input process. When the periods are not near to being related in that way, the expression is often of order  $s$  in size so the output variogram is substantially smaller than the input variogram.

- (3) For an exponential input variogram, say  $V(h) = A[1 - \exp(-B|h|)]$ , the output variogram for lags between 0 and 1 is given by

$$A^{-1}s^2V_O(h) = s - s\exp\left(\frac{-Bh}{s}\right) - \left[\exp\left(\frac{-Bh}{s}\right) + \exp\left(\frac{Bh}{s}\right) - 2\right] \times \\ \left[(s-1)\exp\left(\frac{-B}{s}\right) + (s-2)\exp\left(\frac{-2B}{s}\right) + \dots + (1)\exp\left(\frac{-(s-1)B}{s}\right)\right]$$

This expression can be simplified by taking  $\gamma = \exp\left(\frac{-B}{s}\right)$  and noting that

$$(1 - \gamma)^2 \left[(s-1)\gamma + (s-2)\gamma^2 + \dots + (1)\gamma^{s-1}\right] \\ = (1 - \gamma) \left[(s-1)\gamma - \gamma^2 - \gamma^3 - \dots - \gamma^s\right] \\ = (s-1)\gamma - s\gamma^2 + \gamma^{s+1} \\ = \gamma [s - 1 - s\gamma + \gamma^s]$$

The simplified expression is

$$V_O(h) = As^{-1} \left[1 - \exp\left(\frac{-Bh}{s}\right)\right] \\ - As^{-2} \left[\exp\left(\frac{-Bh}{s}\right) + \exp\left(\frac{Bh}{s}\right) - 2\right] (1 - \gamma)^{-2} \gamma [s - 1 - s\gamma + \gamma^s]$$

The first term  $V_O(h) \approx As^{-1} \left[ 1 - \exp\left(\frac{-Bh}{s}\right) \right]$  generally provides a good approximation. A cruder approximation may be obtained by approximating the input variogram by the linear variogram  $V(h) \approx AB|h|$ . The output variogram is therefore approximately  $V_O(h) \approx s^{-2}ABh$ . This is generally adequate for applications.

This model is useful when output blocks of material are constituted from consistent proportions of the input blocks. For many blending stockpiles, this is an adequate approximation for the middle of the piles but not for the ends.

#### 4 Blending based on realistic geometries

Bed-blending assumes that each reclaim slice includes precisely equal amounts of material from all layers of material stacked. This is a good approximation for some bed-blending systems, but it is not a good approximation for most mineral stockpiling systems. In this section, we concentrate on predicting the performance of stockpiling systems for which reclaim slices include unequal amounts of material from the layers of material stacked.

Suppose that the geometry of a stockpiling system is known and is specified by an  $s \times r$  matrix of masses which relate  $s$  blocks of stacked material to  $r$  blocks of reclaimed material.

$$\begin{array}{cccc}
 m_{s,1} & m_{s,2} & \dots & m_{s,r} \\
 \vdots & \vdots & & \vdots \\
 m_{2,1} & m_{2,2} & \dots & m_{2,r} \\
 m_{1,1} & m_{1,2} & \dots & m_{1,r}
 \end{array} \tag{2}$$

The quantity  $m_{i,j}$  specifies the mass of material from stacked block number  $i$  which is reclaimed in reclaim block number  $j$ .

For each output block we can calculate the proportion which comes from each input block

$$p_{i,j} = m_{i,j} / \sum_i m_{i,j}. \tag{3}$$

Note that the proportions add to unity across an output block in the sense that  $\sum_i p_{i,j} = 1$ , but they do not generally add to unity across input blocks.

If we know the grades of stacked blocks as a vector

$$[g_1, g_2, \dots, g_s]$$

and we are prepared to assume that the input blocks are fairly uniform, then the grades of output blocks can be found by calculating weighted averages. In general, output block number  $j$  has grade  $\sum_i p_{i,j} g_i$ .

Such computations of weighted averages are illustrated in a simple example below.

#### 4.1 Direct computation of variances of differences

Sometimes it is desirable to estimate the amount of variation between output blocks which is likely given a specified pattern of variation between input blocks. People often calculate such estimates by simulation of a large number of possible input grades.

An alternative to conducting such simulations is to compute the variance of differences of interest directly from the matrix of proportions and a variogram which describes the input variation. This is practical when input blocks are taken to be of equal sizes. The calculations below assume that the input variogram is based on a unit lag which is equal to the size of an input block.

Suppose that we are interested in the difference between output blocks  $j$  and  $k$ . The difference in their grades is  $\sum_i (p_{i,j} - p_{i,k}) g_i$ . The difference can be regarded as a weighted average  $\sum_i w_i g_i$  where  $w_i = p_{i,j} - p_{i,k}$ . Note that the weights  $w_i$  add to zero.

Assuming that the grades of all input blocks have the same expectation, the expectation of  $\sum_i w_i g_i$  over the possible variation in the grades  $g_i$  is  $\sum_i w_i E[g_i]$  which is zero. Hence the variance of  $\sum_i w_i g_i$  is

$$\text{Var}[\sum_i w_i g_i] = E[(\sum_i w_i g_i)^2] = \sum_i w_i^2 E[g_i^2] + 2 \sum_{i < j} w_i w_j E[g_i g_j] \quad (4)$$

It can be seen that an alternative, equivalent expression is

$$\text{Var}[\sum_i w_i g_i] = -2 \sum_{i < j} w_i w_j V_I(j - i) \quad (5)$$

where  $V_I(j - i) = \frac{1}{2} E[(g_i - g_j)^2]$  is the input variogram. To check the equality we first note that

$$-2V_I(j - i) = -E[g_i^2] - E[g_j^2] + 2E[g_i g_j].$$

Then we note that the coefficient of  $E[g_i^2]$  in the right hand side of equation (5) is  $-\sum_{j \neq i} w_i w_j$ . Since  $\sum w_i = 0$ , this equals  $w_i^2$  which is the same as in the right hand side of equation (4). Similarly, the coefficient of  $E[g_i g_j]$  is  $2w_i w_j$  in both expressions.

Equation (5) is very useful for allowing variances of differences between output blocks or differences from the stockpile average grade to be efficiently computed. Note that output grade varies more rapidly near the ends of stockpiles than in the middles, so the variation in grade on reclaiming cannot be described by a variogram.

#### 4.2 A simple example to illustrate the computations

Consider the situation illustrated in Figure 1 where four layers of material are stacked in chevrons and the material is then reclaimed by benches. The complex geometry of the end of the stockpile will be ignored and variation within each of the three output blocks of material will also be ignored. Figure 1 shows a cross-section through the stockpile and the numbers given are cross-sectional areas.

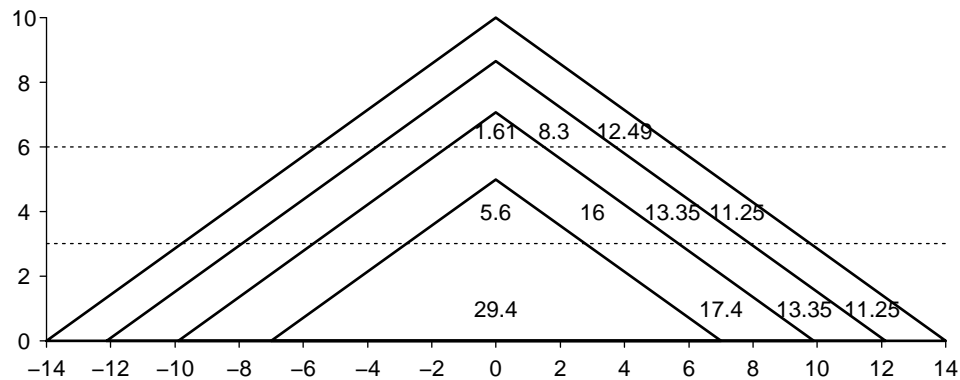


Fig. 1. Cross-section through a chevron-stacked stockpile. The angle of repose of the material is about 35.5 degrees. The dotted lines differentiate reclaim benches and the numbers give components of cross-sectional area. Each of the four layers stacked has a total cross-sectional area of 35 square metres.

The matrix of quantities like expression (2) relating input blocks to output blocks is given by the cross-sectional areas. Note that the top layer is reclaimed

first and corresponds to the first column of the matrix.

12.49	11.25	11.25
8.30	13.35	13.35
1.61	16.00	17.40
0.00	5.60	29.40

The matrix of proportions like those given in expression (3) is calculated by dividing by the column totals.

$$\begin{matrix}
 0.5577 & 0.2436 & 0.1576 \\
 0.3706 & 0.2889 & 0.1870 \\
 0.0717 & 0.3463 & 0.2437 \\
 0.0000 & 0.1212 & 0.4118
 \end{matrix} \tag{6}$$

These proportions are illustrated in Figure 2. They can be used for calculating grades of reclaim blocks. For instance, if the grades of the four input blocks were 13, 15, 14 and 17 then the grades of the three output blocks would be 15.26, 14.41 and 13.81.

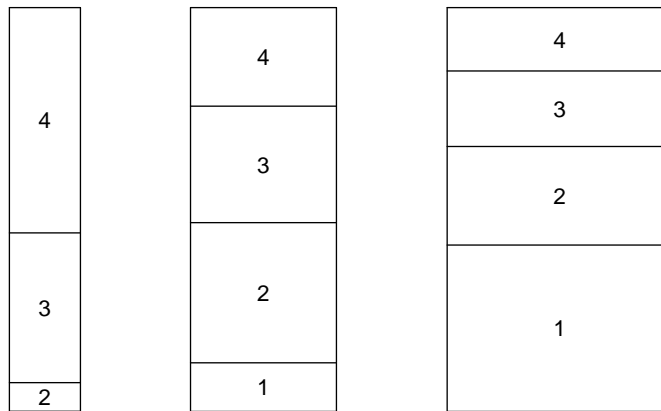


Fig. 2. Stacked bar chart illustrating how material is blended by bench-reclaiming from a chevron-stacked stockpile. Each column represents a reclaim portion. The portions of the column show how much material came from each of the layers on stacking.

Sets of 100 simulations were run with grades of input blocks being taken to be independently normally distributed with unit standard deviation. The average squares of the differences between the grades of first and last reclaim slices were calculated. The results from 10 sets of 100 simulations were 0.46, 0.43, 0.41, 0.37, 0.33, 0.26, 0.38, 0.40, 0.33 and 0.35. The variation between these results is consistent with the expected amount of variation between sets of 100

simulations. In principle, averages of  $n$  simulations would be expected to be distributed like  $\chi^2_{(n)}$  statistics and therefore to have a coefficient of variation of  $\sqrt{2/n}$ . For  $n = 100$  simulations this is about 14%.

Direct computation of the variance of the difference between the grades of first and last reclaim slices from the input variogram  $V_I(h)$  which is 0 if  $h = 0$  and 1 otherwise gave the variance 0.39. This is consistent with the simulations and is faster and more reliable.

Here the variances of the differences from stockpile average grade for the three reclaim portions are 0.2035, 0.0274 and 0.0387. As standard deviations, these are 0.451, 0.166 and 0.197.

### 4.3 *Computing the geometry of stockpiles*

The bulk of the work required to make predictions about how much a blending stockpile would reduce grade variation is in the calculations concerning possible stockpile geometries. The output of such calculations tells us how output blocks are related to input blocks. It can always be regarded as a matrix like the proportions given in matrix (6) and also displayed in Figure 2.

I have used the following methods for various applications.

- Formulae given in Robinson and Ross (1991) describe the geometry of chevron stockpiles for the case when the angle of repose on stacking is approximately equal to the angle of repose on reclaiming.
- A computer model which keeps track of the cross-section of windrow stockpiles can be used for calculating stockpile geometries for various types of reclaimers, but it is not accurate for the ends of stockpiles because the cross-section which is used is appropriate for the middle of a stockpile but not for the ends. This model might be described as a “two-dimensional” model.
- A general purpose stockpile model called “CHASM” which is described in Robinson and van der Touw (1993) can be used for a variety of calculations involving the three-dimensional geometry of stockpiles. It has been used for producing the results given in this paper. It has procedures specifically designed for handling chevron, windrow, conical and shoulder stacking and for tunnel loadout, boom reclaimers and full-width reclaimers. Other methods of stacking and reclaiming can be handled by using sequences of simple procedures. For instance, circular stockpiles can be stacked using a large number of conical stacking operations and by using a procedure which reclaims all material which would rill into a specified quadrilateral.

Other groups of people also have computer software for computing geometry

concerned with stockpiles. The software that I know about is as follows.

- QMASTOR, as described in Keleher, Cameron and Knijnikov (1998) and Cameron, Knijnikov and Manton (1996).
- CTRS, as described in Johnson (1991).
- Software described in a Patent Application by Gamma-Metrics (1999).
- Software used by Robe River Iron Associates which has not been described in any public document.

The relative merits of the various software for stockpile geometry is outside the scope of this paper.

## 5 Stockpiles with uniform stacked layers

We will now consider how much blending can be expected for some stockpiles which have been stacked using material which varies in grade slowly compared to the length of a stacker pass. There are some issues about the blending of material which varies more rapidly in grade which will be deferred until section 6.

The main blending options to be discussed will be chevron and windrow stacking with reclaiming in benches, full-width reclaiming and boom reclaiming. Some less common options will also be briefly mentioned.

The first stockpile to be discussed is a chevron-stacked pile. In cross-section, it is shown in Figure 3. It has 21 blocks. Each block is 2000 tonnes, which is 2500 cubic metres since the bulk density is taken to be 0.8. The angle of repose on stacking is taken to be  $36^\circ$ . The distance between stacking limits is taken to be 200 metres. In cross-section, the pile is about 38 metres wide and 13.8 metres high. A stacker pass has been taken to be 400 tonnes (or 500 cubic metres). The input blocks are a multiple of this, so each layer of material is of uniform grade.

A second stockpile is a windrow-stacked pile, using the same 21 input blocks. Its cross-section is shown in Figure 4. The horizontal lines illustrate the possibility of reclaiming the pile in three benches. The numbers on the diagram indicate the order which the windrows were stacked.

Figure 5 illustrates the matrices of proportions for three rather poor blending regimes. The first, illustrated by the three stacked bars on the left hand side of Figure 5, is a chevron-stacked pile which is reclaimed in benches.

In order to calculate measures of output grade variation, the variability of grade between input layers has been, arbitrarily, taken to be described by the

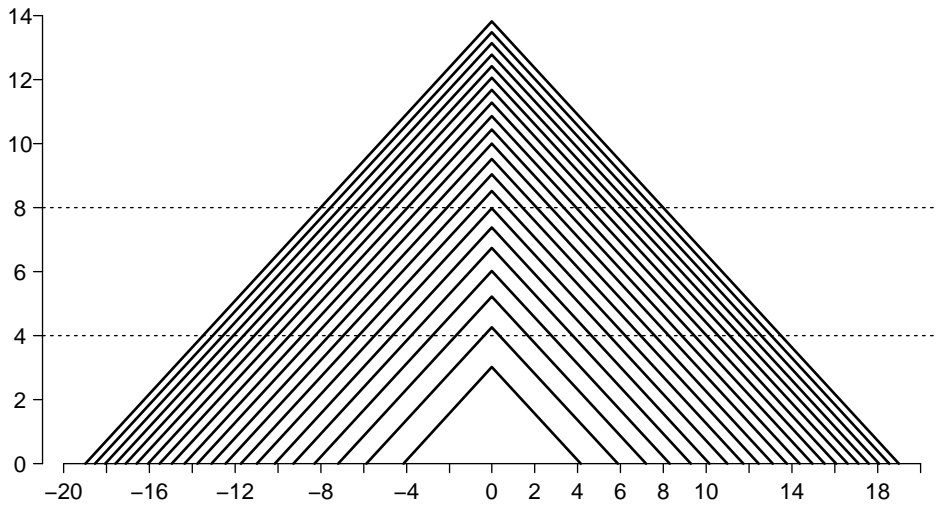


Fig. 3. Cross-section of a chevron-stacked stockpile with 21 stacked layers

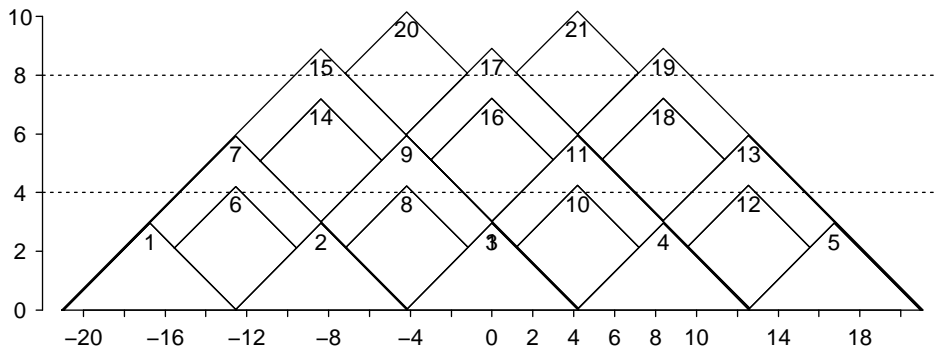


Fig. 4. Cross-section of a windrow-stacked stockpile

variogram shown as the upper line in Figure 6. Batches of 2000 tonnes are taken to be perfectly uniform and the variogram describing half the expected squared difference in grade between the  $i$ th and  $j$ th batches is

$$V(i - j) = 1 - \exp(-0.3|i - j|).$$

### *Reclaiming in benches*

For a chevron-stacked pile reclaimed in benches, blending is illustrated on the left hand side of Figure 5, and the standard deviations of the differences between the average grades of the three benches and the overall average grade are 0.155, 0.016 and 0.027.

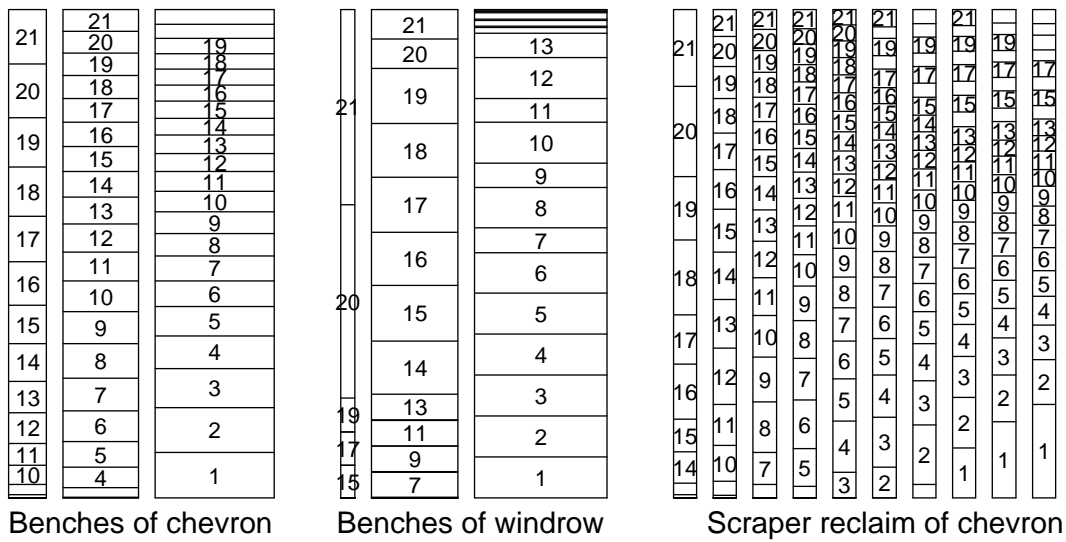


Fig. 5. Stacked bar charts relating input blocks to output blocks for three blending regimes

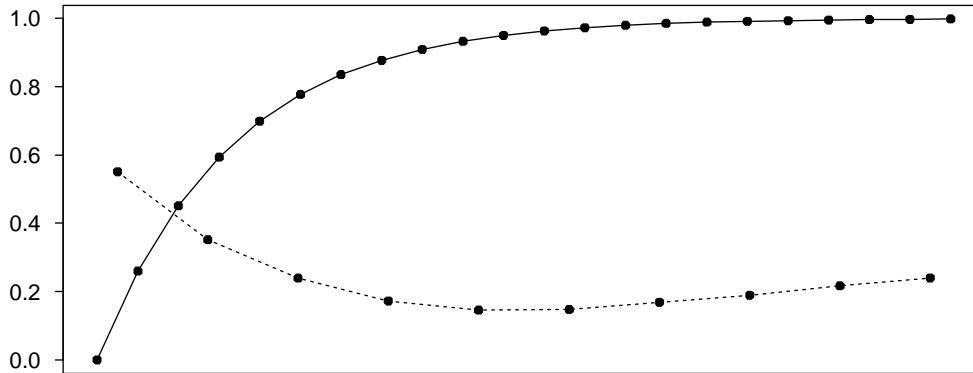


Fig. 6. An input variogram and a measure of output variation. The continuous line shows the input variogram which has been used for most of the examples in this paper. The dotted line shows the standard deviation of the difference between the grades of output blocks and the stockpile average for reclaiming of a chevron stockpile using a scraper reclaimer for ten output blocks.

For a windrow-stacked pile reclaimed in benches as illustrated in Figure 4, the blending is illustrated in the middle of Figure 5, and the standard deviations of the differences between the average grades of the benches and the overall average grade are 0.547, 0.145 and 0.090. The differences between benches are generally much larger for windrow-stacked piles than for chevron-stacked piles, because the benches are further from being an equal mixture of all of the input blocks.

Another very poor blending regime is to use a scraper reclaimer which pivots



stockpile. However, the reclaim blocks which are further from an end of the pile are more similar to the stockpile average.

If the chevron-stacked pile is reclaimed using a full-width reclaimer then the blocks of material on reclaiming each consist of fairly similar proportions of each of the 21 input blocks. Figure 8 illustrates the proportions of the 21 input blocks for the first and last ten output blocks of size 148.7 tonnes for full-width reclaiming of a windrow-stacked pile. The proportions appear to be further from equal than the proportions shown in Figure 7 for a chevron-stacked pile.

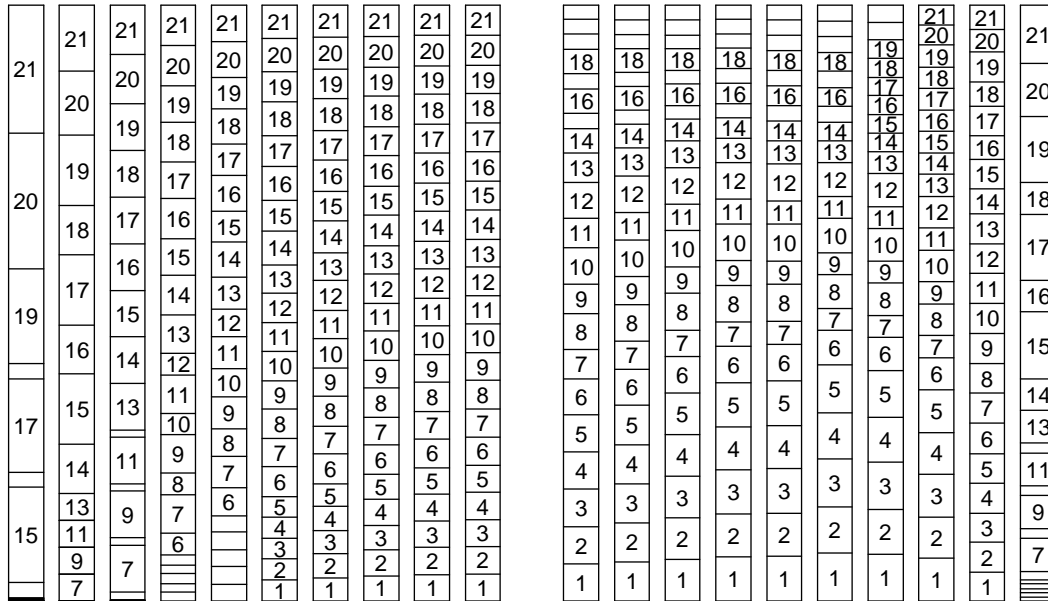


Fig. 8. Stacked bar chart illustrating how material is mixed by full-width reclaiming from a windrow-stacked stockpile. The columns correspond to the first and last ten output blocks out of 336 equal-sized blocks.

As explained in Robinson and Ross (1991), it is possible to stagger the stacking limits for chevron stockpiling in order to reduce the magnitude of these end effects. The recommended staggering is that both stacking limits be moved in the same direction by a distance of the tangent of the angle of repose on reclaiming times half the height of the pile. This staggering guarantees that when the pile is half-reclaimed each individual chevron will be half-reclaimed.

Figure 9 illustrates the composition of the first and last ten output blocks from a staggered chevron pile. These are on average nearer to the stockpile average grade than the output blocks for a standard chevron pile, as will be seen in Figure 13 to be discussed later.

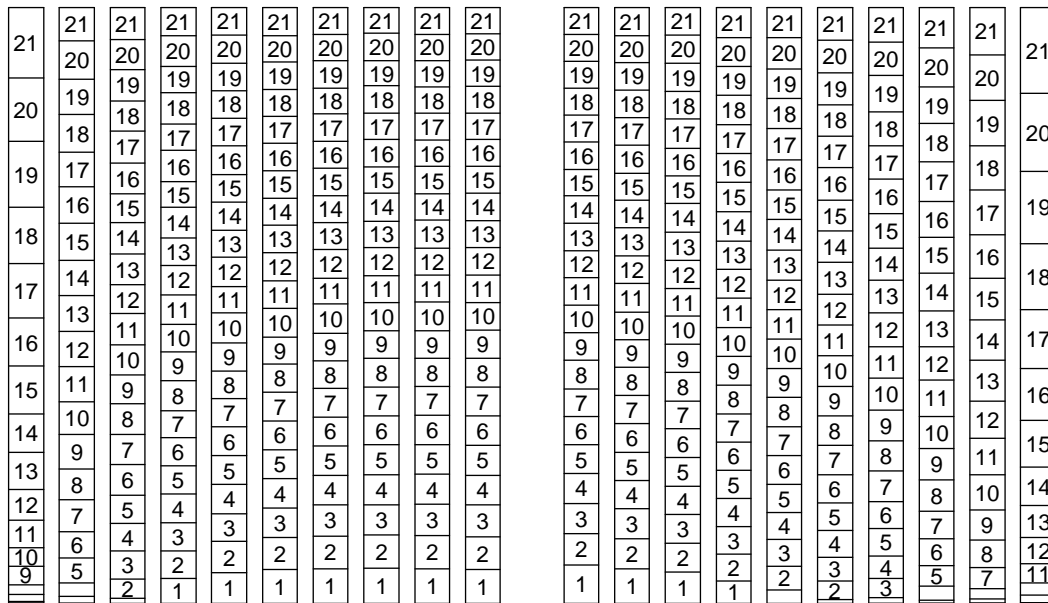


Fig. 9. Stacked bar chart illustrating how material is mixed by full-width reclaiming from a chevron-stacked stockpile when the stacking limits are staggered so that each chevron is half-reclaimed when the pile is half-reclaimed. The columns correspond to the first and last ten output blocks out of 336 equal-sized blocks.

*Boom reclaiming*

Another, widely-used type of blending stockpile is a chevron-stacked pile with a boom reclaimer. Before discussing this blending option, we must first make some assumptions about the mode of operation of the boom reclaimer.

Boom reclaimers generally take a restricted height of material at any time. When boom reclaimers become slightly overloaded, the booms tend to bend making them even more overloaded. For the stockpiles being considered here, it will be assumed that the boom reclaimers will take some material from a bench at a height of eight metres, then take some material from a bench at a height of four metres, then take some material at the level of the base of the stockpile.

If a boom reclaimer takes a large amount of material from a bench before changing benches then the amount of variation in grade between benches will dominate the pattern of variation on reclaiming. We will assume the effect of bench-to-bench grade variation has been minimized by taking portions of the three benches which are as consistent as it possible in practice

Figure 10 illustrates an important aspect of the geometry of boom reclaiming. The plan view of a portion of a stockpile not including an end is illustrated. The horizontal lines show the edges of the stockpile at the base and at the

heights of the benches, here four and eight metres. The curve on the left labelled “1” shows the arc traversed by the point on the bucketwheel which digs into the stockpile as the bottom bench is being dug. Here this curve is approximately 46.5 metres in radius.

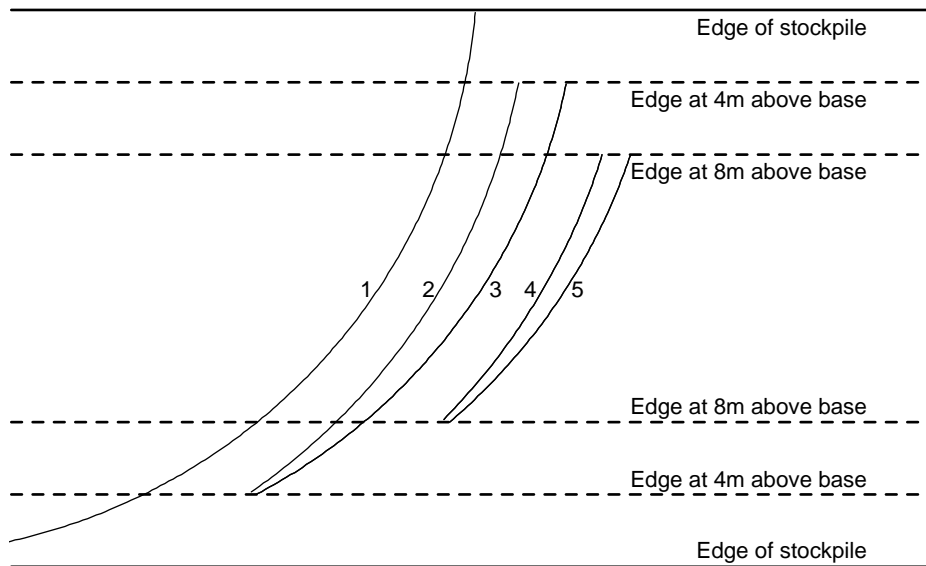


Fig. 10. Plan view of the operation of a boom reclaimer working on three benches

Curve 2 is about 50.5 metres in radius. It shows the upper extremity of the region between the base and the 4 metre bench which rills down to be collected by the boom reclaimer when it is digging at the base of the pile. The increase in radius is, in general, equal to the vertical distance between benches divided by the tangent of the angle of repose on reclaiming.

Curve 3 shows the arc traversed by the bucketwheel as it digs the bench at 4 metres. A small amount of approximately level bench is always left from the previously-discussed line. Note that this approximately level surface is wider on the side of the bench near to the reclaimer than on the other side.

Curve 4 shows the upper extremity of the region which rills down to the boom reclaimer when it is digging at 4 metres above the base of the pile. Curve 5 shows the arc traversed by the bucketwheel as it digs the bench at 8 metres.

Figures 14, 15 and 16 provide perspective views of the same aspect of boom reclaimer geometry. Those figures show reclaiming of a windrow-stacked pile.

In order achieve the plan shown in Figure 10, the reclaimer must finish reclaiming the middle and bottom benches at positions displaced 6 and 13 metres from its position when it finished reclaiming the top bench.

For boom reclaiming from our chevron-stacked pile, Figure 11 shows the composition of the first and last 10 output blocks of the same size as were discussed

for a standard chevron pile. Similarly, Figure 12 shows the composition of the first and last 10 output blocks for boom reclaiming of our windrow-stacked pile. It can be seen from the greater variation in the sizes of the plotted regions that the proportions of the input blocks are not as consistent as for full-width reclaiming from chevron-stacked piles.

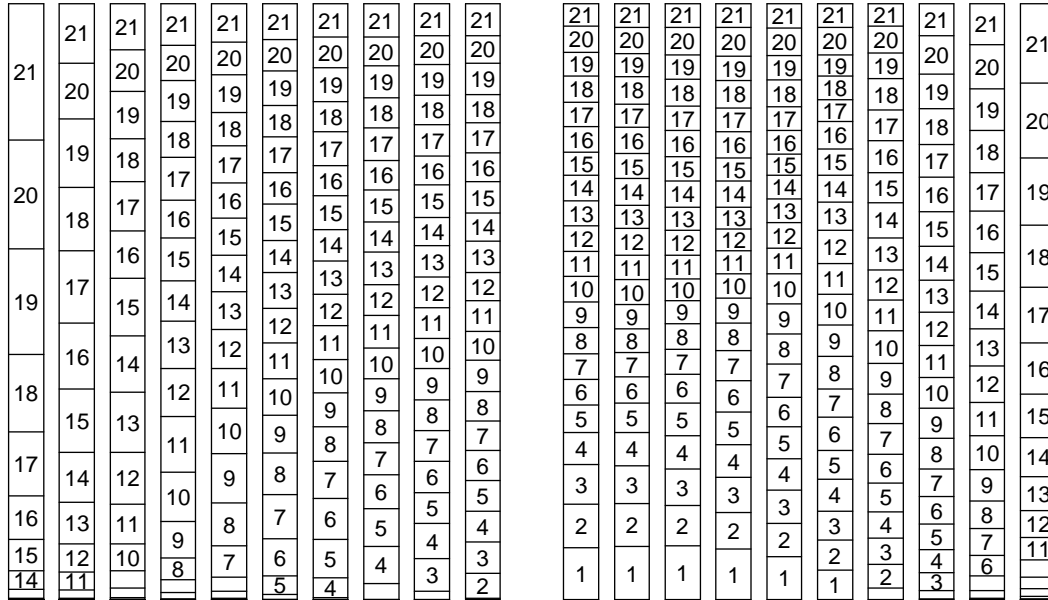


Fig. 11. Stacked bar chart illustrating how material is mixed by boom-reclaiming from a chevron-stacked stockpile. The columns correspond to the first and last ten output blocks out of 336 equal-sized blocks.

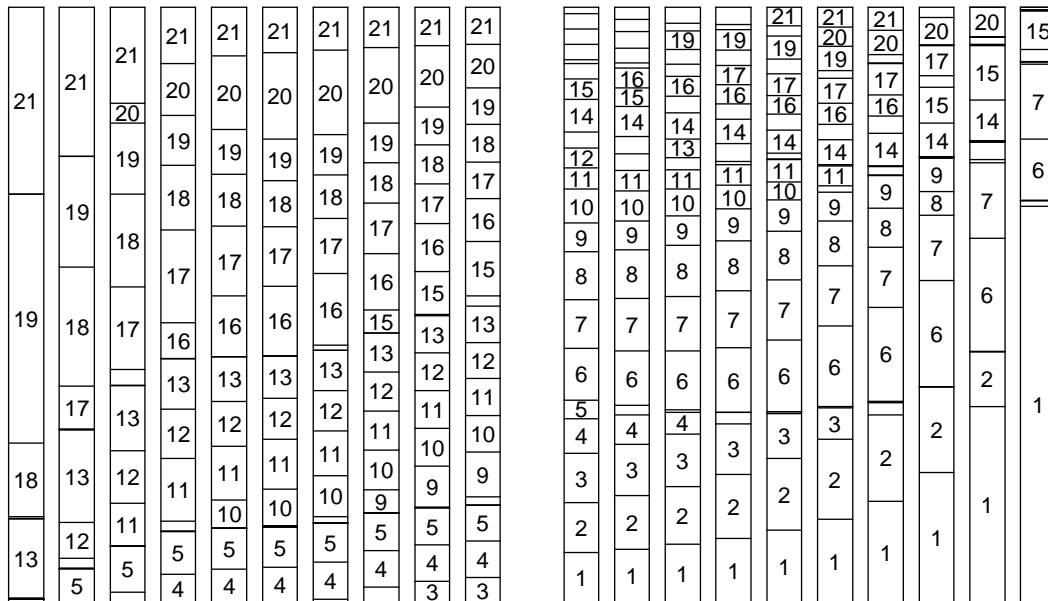


Fig. 12. Stacked bar chart illustrating how material is mixed by boom-reclaiming from a windrow-stacked stockpile. The columns correspond to the first and last ten output blocks out of 336 equal-sized blocks.

Figure 13 compares the options for blending stockpiles which have been discussed. The horizontal axis shows the reclaim block being discussed. Only the first and last ten blocks of size  $\frac{1}{2}$  an hour of material are shown. The vertical axis shows the standard deviation of the departure from stockpile average grade for the input variogram illustrated in Figure 6.

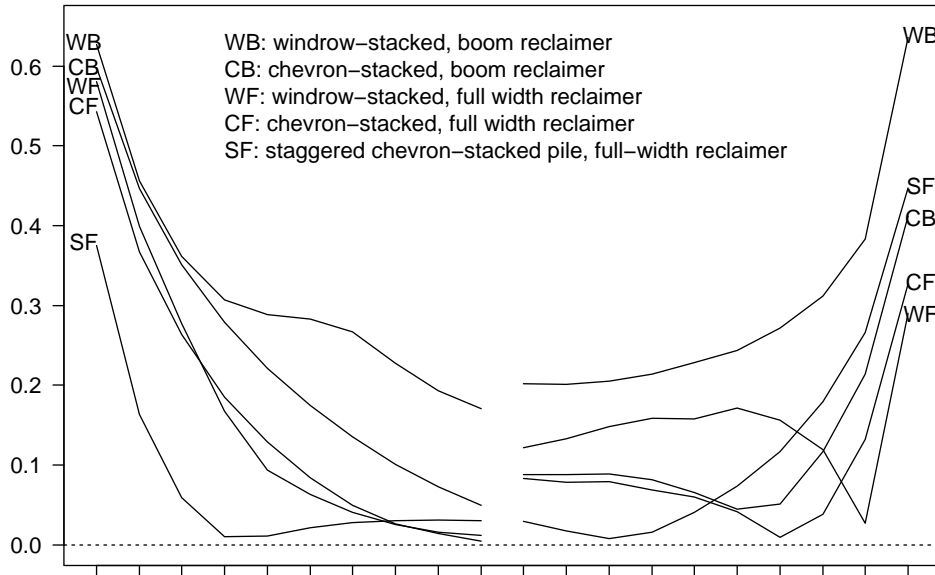


Fig. 13. Standard deviations of the departure from stockpile average grade for several different stockpiling options

It can be seen that blending is worst for a windrow-stacked pile and a boom reclaimer, better for a chevron-stacked pile and a boom reclaimer and better again for a chevron-stacked pile and a full width reclaimer. A chevron-stacked pile and a boom reclaimer is better than a windrow-stacked pile and a full width reclaimer at the start of reclaiming, when they are both at their worst, but is worse at some other times. Staggered chevron stacking is better than standard chevron stacking at the start of reclaiming, but is not as good at the end of reclaiming.

#### *Checking realism of models*

Figures 14, 15 and 16 illustrate one type of figure that could be produced to satisfy users of this type of technology that the models are realistic. They are perspective views of the windrow-stacked stockpile after it has been partly reclaimed by a boom reclaimer. The position of the viewer has been moved in an anti-clockwise direction between the three views.

It is possible to discern the peaks of the windrows, with their triangular cross-sections. The crescent-shaped areas of horizontal flat benches at 4 and 8 metres

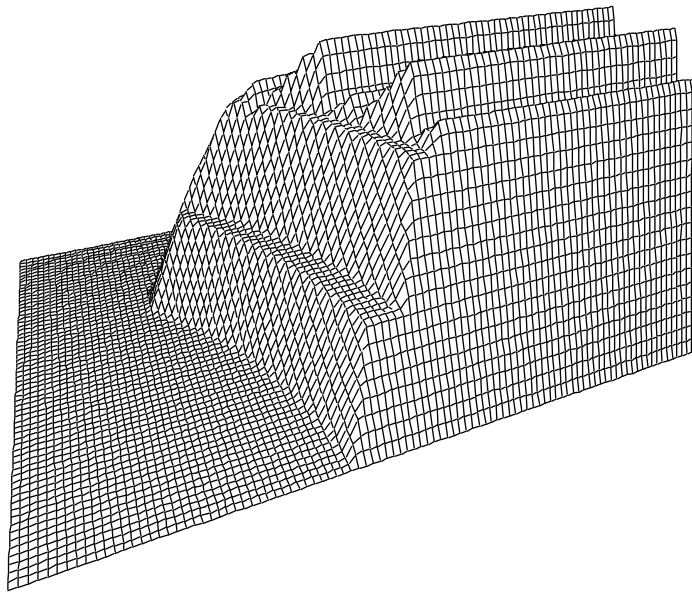


Fig. 14. First perspective view of a windrow-stacked stockpile which has been partly reclaimed using a boom reclaimer

from the base can also be seen. A curved region to the left of the face exposed by reclaiming is visible in Figures 15 and 16. It is part of the end of the stockpile which was curved at the completion of stacking has not yet been removed by the reclaimer.

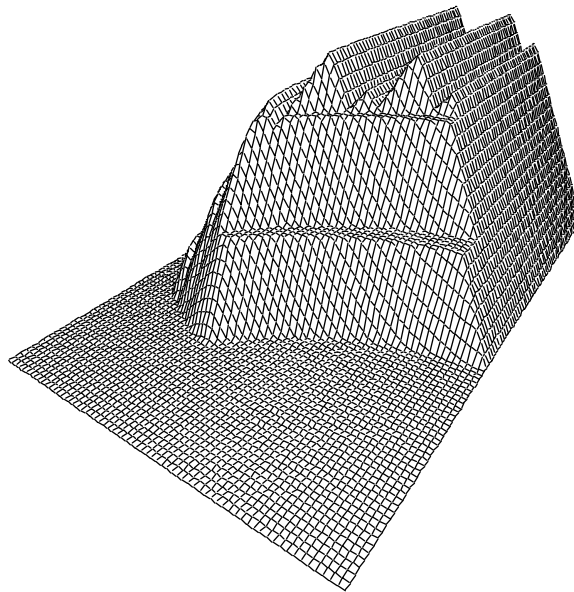


Fig. 15. Second perspective view of a windrow-stacked stockpile which has been partly reclaimed using a boom reclaimer

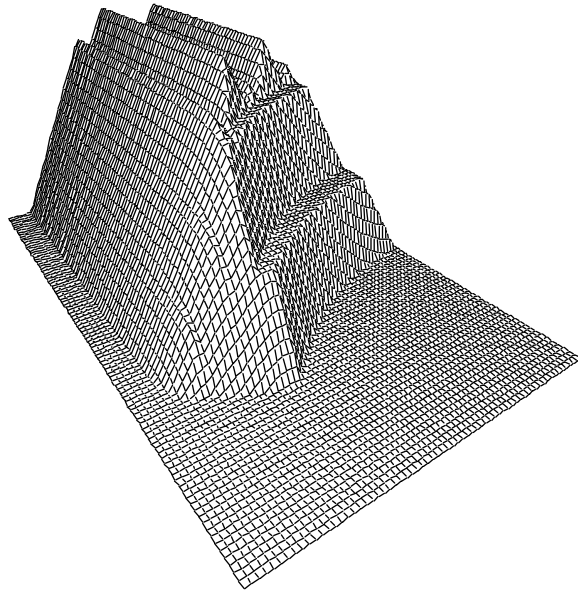


Fig. 16. Third perspective view of a windrow-stacked stockpile which has been partly reclaimed using a boom reclaimer

### *Remarks about segregation*

Given that windrow stackers are more expensive than chevron stackers, these comparisons suggest that chevron stacking ought generally to be preferred. One reason often cited for preferring windrow stacking to chevron stacking is that it reduces the likely amount of segregation. This is indeed relevant, but I have seen little data on the amount of segregation which actually occurs. All the stockpiles that I have ever seen have some very large particles near to the base. This shows that some segregation occurs but it does not quantify how much segregation occurs or tell us whether it would substantially affect within-stockpile grade variation.

## **6 Stockpiles with non-uniform stacked layers**

One circumstance when stacked layers are far from uniform is when a stockpile is used to blend material of substantially different grades. If the material arrives in lots such as trainloads which are larger than the amount of material in a stacker pass, then we need to be aware that these lots do not necessarily terminate at ends of stacker passes.

Consider a trainload of material A placed onto a stockpile, with no particular effort being made to check up on the position and direction of travel of the stacker at the ends of this load. Usually, the stacker will not be very near an

end of the pile when stacking commences and will not be very near an end of the pile when stacking is completed. Hence there is approximately probability  $\frac{1}{2}$  that there will be the same number of layers of that material at the two ends of the pile and probability  $\frac{1}{2}$  that one end will have two more layers than the other end.

The influence of this effect on the amount of within-stockpile grade variation can be calculated by using bed-blending theory, with an input variogram which takes into account the probability that a step change in grade occurs (due to a change from one input lot to another) as a function of lag. This approach is adequate for calculating an estimate of the change in variability which would be likely to be achieved by changing stacker tramming speed.

Another circumstance when stacked layers are far from uniform is when a stockpile is built from truckloads which are smaller than stacker passes. This is a potential problem if there is likely to be cyclical variation in grade which has a period which is simply related to the length of a stacker pass.

For instance, suppose that truckloads of average size 100 tonnes are being stacked onto the chevron stockpile that we discussed earlier, with 400 tonnes being stacked per stacker pass. Further suppose that the trucks are being served by two digging machines. It is likely that the numbers of trucks associated with each digging machine will be set by operational planning staff in such a way that trucks from the two digging machines arrive alternately. Such alternation might happen to be maintained perfectly for several hours, because the digging machine which is moving material slightly faster is not fast enough to ever get two truckloads dumped between a pair of trucks from the other digging machine. This would mean that there was strong cyclical variation with period 200 tonnes.

This situation was simulated with the stacker starting at a point 50 metres into a stacker pass, stacking 500 truckloads having sizes which were normally distributed with mean 100 tonnes and standard deviation 2 tonnes. It is natural to generate such simulated scenarios as part of planning a new mine. My purpose here is to illustrate the point made in section 4 of this paper that expected variation can be calculated directly using geometry and an input variogram.

Two alternative input variograms are illustrated in Figure 17. When truckloads come from the same source, it is assumed that the variogram is  $1 - \exp(-0.3|h|)$  for a lag of  $h$  trucks. When truckloads come from different sources, it is assumed that the variogram is exactly 10, no matter what the separation between the trucks.

The two input variograms differ in that they make different assumptions about how quickly the probability that two trucks have come from the same source

reverts to 0.5 as the trucks get further apart in the sequence. If truck 1 comes from source A then it is very likely that truck 2 comes from source B and truck 3 comes from source A. However, by truck 30 or 100 it becomes more and more likely that the sequence will have been broken, perhaps by a shift change or a staff break.

Both of the input variograms are larger for lags 1, 3 and 5 than for lags 2, 4 and 6 because trucks separated by an even number of trucks are likely to be similar because they have come from the same source. The input variogram on the left shows very persistent alternation of trucks while the variogram on the right shows very little persistence.

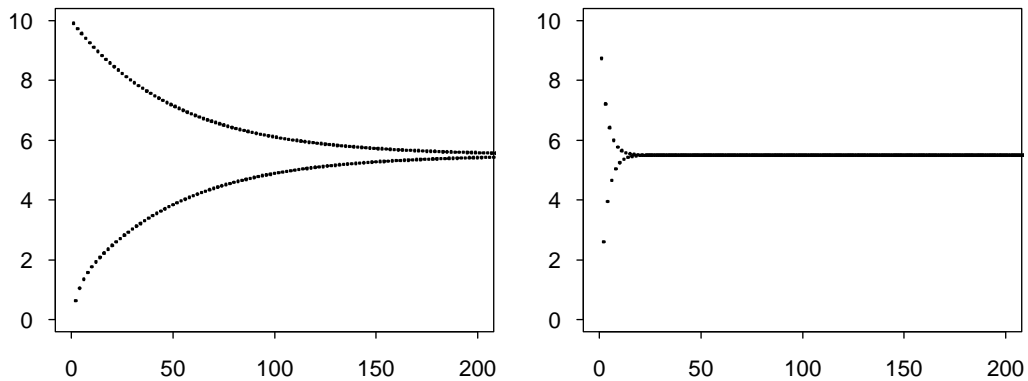


Fig. 17. Two input variograms based on truckloads of ore arriving from two different sources. They are only plotted for lags which are a whole number of truckloads. The left hand plot gives the variogram for more persistence in the pattern of alternation of trucks between the two sources.

Figure 18 shows the standard deviations of the expected variation between stockpile average grade and the grade of a 1/20 portion of the stockpile as reclaimed. There are two lines corresponding to the two input variograms. The upper line on Figure 18 corresponds to the situation when strict alternation of trucks is more persistent. (i.e. the left hand variogram in Figure 17 which takes longer to converge to 0.5).

This type of scenario should be avoided if possible, but a situation like this is unlikely to have such dramatic effects in practice because the truck cycle time is less likely to be constant than is the pattern of alternation between the two digging machines, but the cycle time of the stacker is likely to be very consistent. Hence material from the two source is likely to be distributed over the length of the stockpile even while trucks from two sources alternate perfectly.

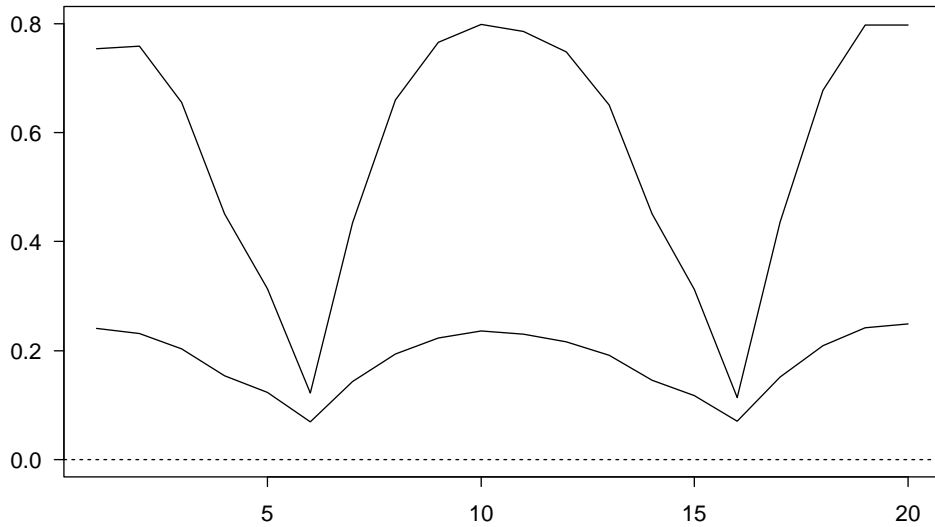


Fig. 18. Standard deviations of discrepancies between stockpile average grade and the grade of reclaim portions of the stockpile. The upper line is for a situation when alternation of trucks between two sources of material is more persistent.

## 7 Variation induced by reclaimers

Blending stockpiles often do a good job of reducing variation between large output blocks, but they often produce output which is quite variable in grade with a cycle which follows the cyclical behaviour of the reclaimer.

When a chemical or metallurgical plant is being fed from a stockpile it is often desirable that a reclaimer be used which mixes material from the entire exposed face of the stockpile. However, some plants are not sensitive to short-term variation and some materials-handling operations do not feed such plants and are only concerned to reduce variation between quite large lots, such as shipments.

The amount of short-term variation induced by a reclaimer can be estimated in much the same way as the variation between larger output blocks was estimated. Figure 19 shows how the composition of material reclaimed is likely to vary as a reclaimer such as the bucketwheel of a bridge reclaimer moves across the face of the chevron-stacked stockpile. The distance across the face was divided approximately into 20 regions for this calculation. They turned out to have unequal amounts of material in them partly because the discrete horizontal grid used in modelling of the stockpile provides barely adequate precision for this purpose.

It is easy to see that, as would be intuitively expected, the late-stacked layers

are over-represented when the reclaimer is near to either side of the pile and the early-stacked layers are over-represented when the reclaimer is near to the middle of the pile.

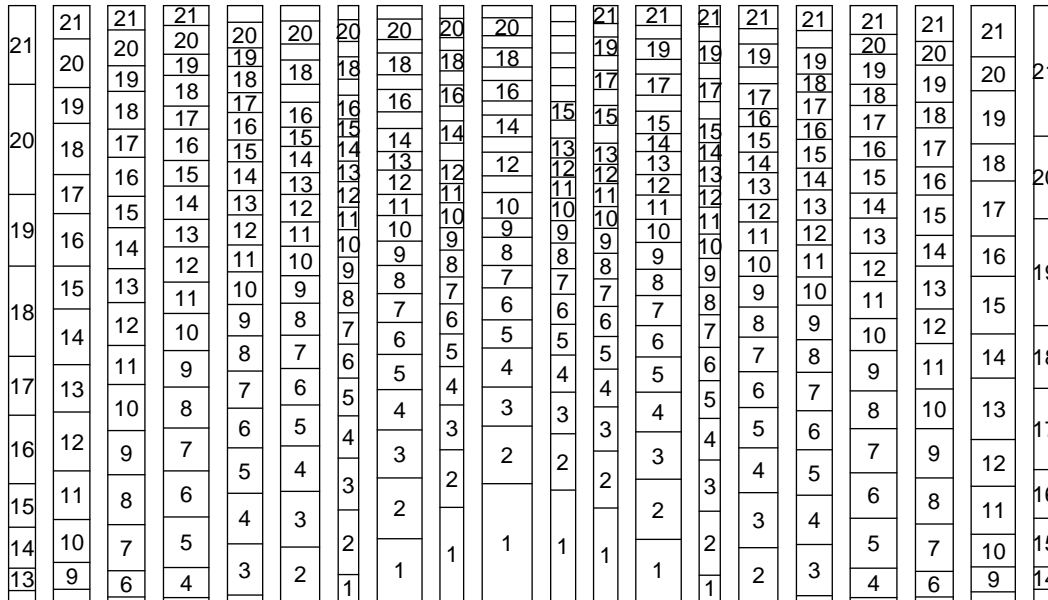


Fig. 19. Stacked bar chart illustrating variation likely to be induced by the motion of the bucketwheel across the face of the stockpile when reclaiming from a chevron-stacked stockpile with a bridge reclaimer.

Figure 20 shows how the composition of reclaimed material varies as a reclaimer such as the bucketwheel of a bridge reclaimer moves across the face of the windrow-stacked stockpile. The distance across the face used the same 20 regions as were used for the chevron-stacked pile. The faces of the two stockpiles have quite different shapes so the amounts of material in the regions are different. (The amounts are proportional to the widths of the columns in the charts.) It is clear that the average number of layers being mixed is lower than for the chevron-stacked pile.

Figure 21 shows the standard deviation of the expected variation between stockpile average grade and the grade of a reclaim portion of the stockpile as the reclaimers move across the face of the piles. The lower line is for the chevron-stacked pile and the upper line is for the windrow-stacked pile. The input variogram assumed was again that shown by the continuous line in Figure 6.

It is generally true that there is more reclaimer-induced variation with a windrow stockpile than with a chevron stockpile.

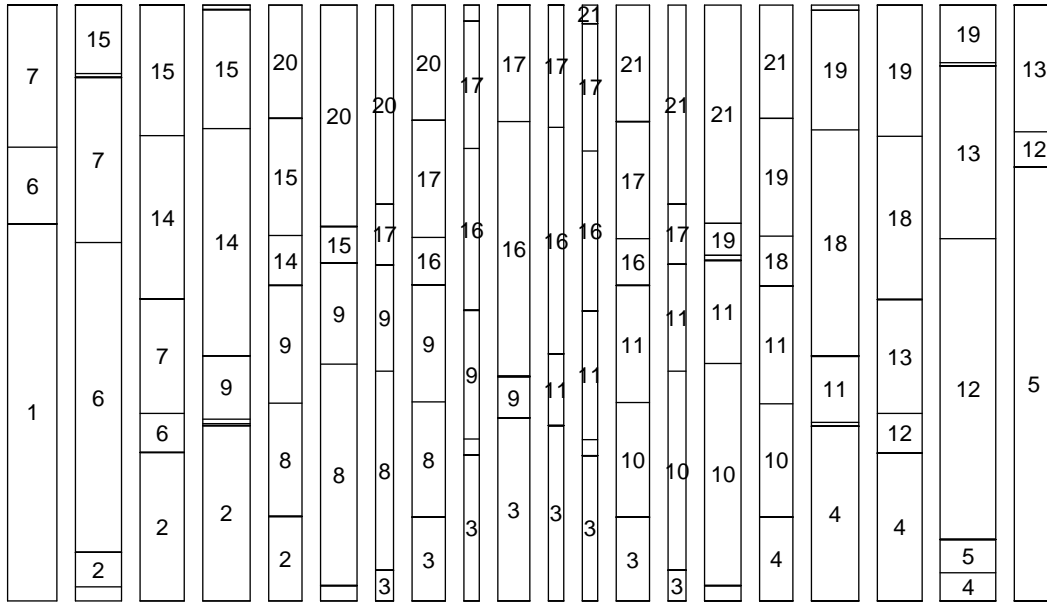


Fig. 20. Stacked bar chart illustrating variation likely to be induced by the motion of the bucketwheel across the face of the stockpile when reclaiming from a windrow-stacked stockpile with a bridge reclaimer. The 20 regions across the face of the pile are geometrically the same as for the chevron-stacked pile but contain different amounts of material.

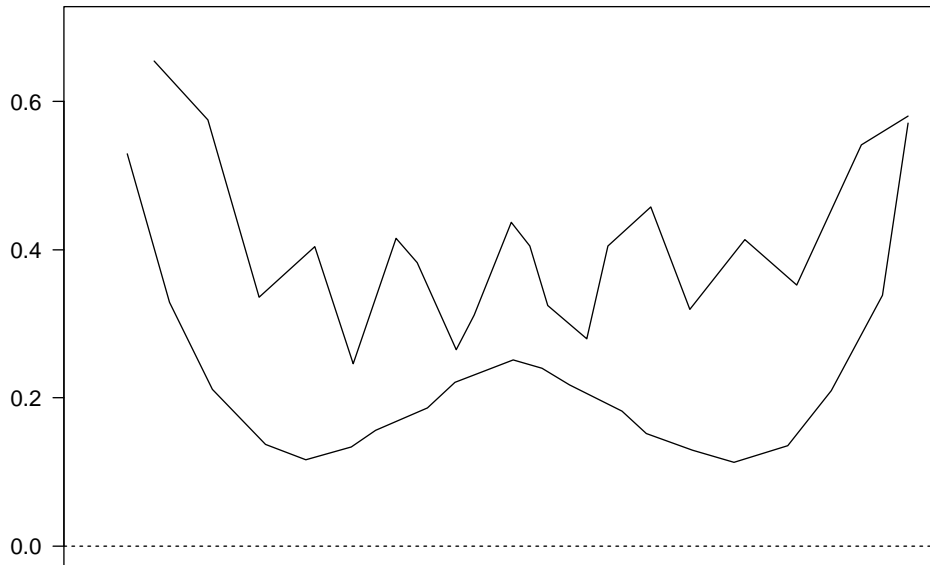


Fig. 21. Variation induced by reclaiming expressed as standard deviations of discrepancies from stockpile average grade. The lower line is for a chevron-stacked pile and the upper line is for a windrow-stacked pile.

## 8 Blending in a wider context

Predictions about the variation which will happen in a materials handling operation are part of what David (1977) calls “fluctuation prediction”. See Robinson (2003) for a discussion of why such prediction should be regarded as important. Because the blending function of stockpiles is often not as commercially important as the buffering and targeting functions, poor predicted performance at blending may be acceptable in the context of the other two functions.

### *How much variation is acceptable?*

An important part of the context in which the amount of blending might be predicted is that it is necessary to ask the question “How much variation is acceptable?”.

To answer this question requires that the tolerance of processes be understood.

- We need to look at the sequence of processes which material must undergo in order to see what time or tonnage scales matter to the most critical processes.
- We need to know which characteristics of the material are important.
- We need to know how much variation in those characteristics is acceptable. Sometimes this may be specified by giving specification limits. For other processes it is crucial that the rate of change of material characteristics be constrained.

### *Other contributions to the pattern of variation*

Blending stockpiles are not the only component of materials handling systems which affect the pattern of variation of material characteristics.

- For mining operations, the incoming variation depends on the pattern of variation in the ground and the mining regime chosen. For other operations, incoming variation depends on choices made about suppliers and contracts.
- Some blending occurs in mills, bins and storage tanks. However, it may be a sensible conservative approximation to assume that they do not blend material.
- Material segregation may occur in bins.
- Any attempt to split a stream of material into nominally identical sub-streams is likely to introduce variation.

## 9 Applications

Some of the actual or proposed mining operations for which I have attempted to predict the amount of blending which would occur in a stockpile are as follows.

- For a proposed coal mine which was intended to feed a power station, I attempted to predict the amount of hour-to-hour variation of the sulphur dioxide emissions from the power station.
- For two proposed nickel-mining operations, I have modelled some options for blending stockpiles with a view to helping ensure that the amount of variation will be acceptable to the extraction plants. In one of these projects, I considered alternative options for reclaimers.
- For a current nickel-mining operation, I have considered the amount of variation likely to be currently occurring in bins and mills and the amount of variation which would be likely to occur if blending stockpiles were installed.
- For an upgrade to a copper & uranium mine, I modelled chevron-stacked piles, two sizes of circular stockpiles and several variations of tunnel loadout piles. A tunnel loadout option was eventually chosen although its expected blending performance was the poorest, because the equipment to be used was thought to be much more reliable.
- For refurbishment of a tunnel loadout stockpile for lump iron ore, I modelled the blending performance of a few different stacking regimes.
- I have estimated the effect of changing stacker tramming speeds at three different points in iron ore mining operations.

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