Non-parametric option pricing models

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Non-parametric option pricing methods are techniques that attempt to price options relying on as less parametric model assumptions as possible. Some of them do not involve any financial theory but estimate option prices using an inductive process applied to vast quantities of historical data. Others use some financial assumptions to ensure rational pricing.

Traditional parametric models are based on assumption of specific parametric form for stochastic process of the option underlying assets. The parameters are calibrated using some criteria, e.g. best or exact fit to prices of some instruments. Evolution of the famous Black-Scholes model demonstrates the problems with the traditional approach. That is, Black-Scholes model presented in 1973 and pioneered parametric option pricing was based on a number of assumptions like continuous diffusion, constant volatility, constant interest rates, Normal distribution of returns, etc. Since then, a wide range of models, e.g. stochastic volatility or jump diffusion models, relaxing some of the assumptions have been developed and are being developed to achieve more realistic modelling. Often these models produce parameters inconsistent with historical data time series and often are outperformed by simple models. As attempt to overcome the problems with traditional parametric models a new class of non-parametric methods was introduced to price options using as less model assumptions as possible. In general, these methods can be classified into "**Model free non-parametric methods**" and "**Rational non-parametric methods**".

Model free non-parametric methods do not rely on any financial theory and estimate option prices inductively from vast quantities of historical data. Suppose a call option can be priced via formula $Q(t) = \Psi(S(t), K, T, \vec{a})$, where $Q(t)$ and $S(t)$ are the option price and its underlying asset value at time *t*, K is strike, T is maturity and $\vec{a}$ are some parameters affecting pricing (they can include asset prices prior to *t* to incorporate non-Markovian framework). In general, model free non-parametric methods attempt to estimate the pricing functional $\Psi(.)$ using historical data for option prices. Usually, they are based on estimation of relationships such as $Z_i = h(X_i) + \varepsilon_i$, $i = 1,...,n$, where Z_i, X_i are some observed random variables and ε_i is a random noise. Many techniques based on different assumptions have been developed. Among them are:

- **Kernel-based smoothers** that use some procedures of averaging data to reduce error, see for example Hardle (1993), Ait-Sahalia et al. (1995). It was shown for the case of American option pricing that the kernel methods can be more accurate than traditional parametric methods.
- **Genetic programming methods** based on consideration of some rules/operators and their combinations applied to the data. The worst performing rules are eliminated iteratively and surviving rules are mutated by randomly selecting and crossing-over

bits from others. For example, the method for pricing American put considered by Keber (1998) is based on heuristic walk over the tree of rules to find the best pricing rule.

- **Artificial Neural Networks (ANN)** that estimate prices by learning process (network training) using vast historical data. It was demonstrated that ANN can provide accurate option pricing in many markets, see for example Malliaris and Salchenberger [1993], Hutchinson et. al. (1994), Carelli et al (2000). The main problems encountered with ANN option pricing are inability to model deep-in-the-money and shortest maturity options.

Pure non-parametric methods rely on availability of high frequency data and thus these methods are suited for liquid markets while difficult to use for pricing OTC instruments. However, typically, there are several option prices observed at the same time (e.g. calls for different strikes and maturities). In this situation, using model free non-parametric methods raises difficulties because the methods are disconnected from financial arguments and cannot capture restrictions implied by the arbitrage. For example, the call option with strike K_1 should not be cheaper than call option with strike $K_2 > K_1$ and the same maturity.

Rational non-parametric methods incorporate relationship between option price and its terminal payoff into pricing scheme using theory of equivalent martingale measures. In short, the theory implies that in the absence of arbitrage, discounted instrument price is a martingale under some risk neutral probability measure. In particular, in the risk-neutral world all assets must earn the same return and the fair option price is a martingale. That is, the option price is the expectation of the discounted option payoff in respect to the risk-neutral density function. Adding risk-neutral market constraints improves pure non-parametric methods, see e.g. Barucci et al. (1997).

In general, the rational non-parametric methods attempt to estimate the risk neutral density. It is well known, that the risk-neutral distribution of the option underlying asset can be estimated from vanilla prices. For example, in the one dimensional case, the risk neutral distribution can be obtained from the second derivative of the call price in respect to strike. As call prices are available for a limited number of strikes only, the various interpolation techniques can be used to estimate the final distribution via the derivative. Alternatively, some flexible parametric form of the distribution is assumed and fitted to the observed vanilla prices, see for example Jarrow and Rudd (1982) or Rubinstein (1998).

Other rational non-parametric methods assume some conventional model, e.g. Black-Scholes framework, where volatility is deterministic function of the underlying and time. Then the volatility is estimated either parametrically or non-parametrically to match various market option prices. Famous examples are: Implied Binomial Trees by Rubinstein (1994), local volatility model Dupire (1994) and Derman Kani (1994). These methods can be used to price exotic path dependent options consistently with simpler instruments.

Further Reading

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See also: Calibration of option pricing models; Neural networks (artificial neural networks); Fuzzy logic; Genetic algorithms.