Ladder options

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A ladder option is an option that locks in gains once predetermined price levels are reached by the underlying security. The set of predetermined price levels is called "ladder" and the price levels in the set are called "rungs". Typically, the rungs are equally spaced above the option strike with the distance equals to some percentage (e.g. 10%) of the strike. For example, assume that today's underlying asset price is \$19 and consider the ladder call option whose strike is \$19 and rungs are \$19.5, \$20, \$20.5, \$21 and \$21.5. At maturity, the option pays the difference between the highest rung reached during the option life and strike or expiries worthless if no rung has been reached. The dynamics of the process is as follows. If the asset price reaches \$19.5 rung then \$0.5 profit is locked in regardless subsequent movements of the asset price. If afterwards, the asset price will increase and reach \$20 rung before the expiry then extra \$0.5 gain will be locked-in and so on. If the maximum asset price during the option life is \$20.7 then the highest rung reached is \$20.5 and the payout at expiry is \$20.5-\$19=\$1.5.

The ladder option can be seen as a lookback option, where the maximum M is not the highest price $S_{\max}$ reached by the underlying asset during the option life but a step function of $S_{\max}$. That is: $M(S_{\max}) = 0$ if $S_{\max} < K_1$; $M(S_{\max}) = K_n$ if $S_{\max} \geq K_n$; and $M(S_{\max}) = K_i$ if $K_i \leq S_{\max} < K_{i+1}$ for i=1,2,..,n-1, where $K_1, K_2, ..., K_n$ are predetermined price levels. Obviously, the ladder option is cheaper than corresponding lookback option. Also, the ladder option can be represented as a series of barrier options with the trigger levels set at the rungs. For example, the above considered ladder option can be represented as five one-touch barrier options with the trigger levels set at \$19.5, \$20, \$20.5, \$21, and \$21.5 correspondingly. Each of these barrier options pays \$0.5 at maturity if its trigger level is reached by the asset during the option life or expiries worthless otherwise.

The ladder put option is structured with the rungs defined below the strike. Its payout at the expiry is the difference between the strike and the minimum rung reached. The examples of more complicated versions of the ladder option are: Asian ladder, where the rungs are specified for the asset average; Quanto ladder, where the rungs are specified in one currency while payment is made in another one. Sometimes the ladder option is structured to pay the greater of: the highest rung reached less strike; or the asset price at expiry less strike floored to zero.

The ladder option can be viewed as the option that periodically resets its strike and locks the profit between the new and old strikes when the underlying asset trades through the specified rungs. Once a rung is reached, the gain is guaranteed even if the underlying asset falls back. If other rungs are reached, the corresponding gains are guaranteed at each level. Ratchet option has a strike resetting feature similar to the ladder option. However, in the case of ladder option strike resets when the asset hits predefined price levels while in the case of ratchet option strike resets at some predetermined dates. Due to

gain lock-in mechanism, the ladder option is also called as “Lock-In” or “Step-Lock” option. A ladder option without a cap or a limit on the number of ladder rungs is known as “Infinite Ladder” option. This instrument has an open-ended lock-in feature. In the limit of small distance between the rungs, infinite ladder option approximates a lookback option (every time the stock price increases, the gain is locked in). Ladder options appear quite often in the market of foreign exchange derivatives. Also, they are known in the equity and fixed income markets.

Pricing of the ladder option is similar to the pricing of lookback or barrier options. Closed form solutions can usually be found assuming Black-Scholes model for the underlying asset with constant volatility and yields. However, ladder option is path-dependent and thus is sensitive to the forward volatility. To price the option consistently with the observed implied volatilities at different maturities, Black-Scholes model with time-dependent volatility and rates can be used. In this case, closed form solutions are not available and numerical techniques such as Monte Carlo or finite difference will be required to value the option. More accurate pricing should take into account dependence of the implied volatility on strike, so-called volatility smile phenomenon. The local volatility models (e.g. Dupire (1994), Derman and Kani (1994)), stochastic volatility models, jump diffusion models or their combinations can be used for pricing of the option consistent with the observed implied volatilities across all available strikes and maturities. The ladder option can be expressed via barrier options, and thus, its risk management can follow the same rules. Static replication can be used for hedging ladder option but can be risky if pricing model is not realistic.

Ladder options are applicable for risk-averse option buyers as profits are progressively locked in without losing the option position. The buyer of the ladder option locks-in his gain each time a rung is reached during the option life. This gain is locked regardless of the consequent market movements. At maturity, the investor receives the sum of the locked-in gains. Not surprisingly, this structure is popular with long term investors seeking protection against market volatility. The option is considered as complex defensive because it is less risky than traditional options, the profits are locked in as underlying asset performs, and there is no need to constantly watch the underlying market levels. Of course, as a result of these benefits ladder option is more expensive than corresponding vanilla option. In volatile markets, the ladder option is more advantageous than the vanilla option due to the gain lock-in feature, but it has a little advantage in the markets with trends.

Further Reading

Derman, E. and I. Kani (1994), Riding on a smile, *Risk*, 7 (2), 32-39.

Dupire, B. (1994), Pricing with a smile, *Risk*, 7 (1), 18-20.

Howard, K. (1995), Cliquet Options and Ladder Options, *Derivatives Week*, May 95, p.7

Wilmott, P. (2000), *Paul Wilmott on Quantitative Finance*, John Wiley & Sons.

See also: Barrier options; Lookback options; Path-dependent options; Ratchet options; Volatility skews and smiles.