Pavel V. Shevchenko
Quantitative Risk Management

CSIRO
Mathematical & Information Sciences

www.csiro.au

Bridging to Finance

Conference "*Quantitative Methods in Investment and Risk Management: sourcing new approaches from mathematical theory and the real world*"
Melbourne Centre For Financial Studies, 20th September 2007.

CSIRO

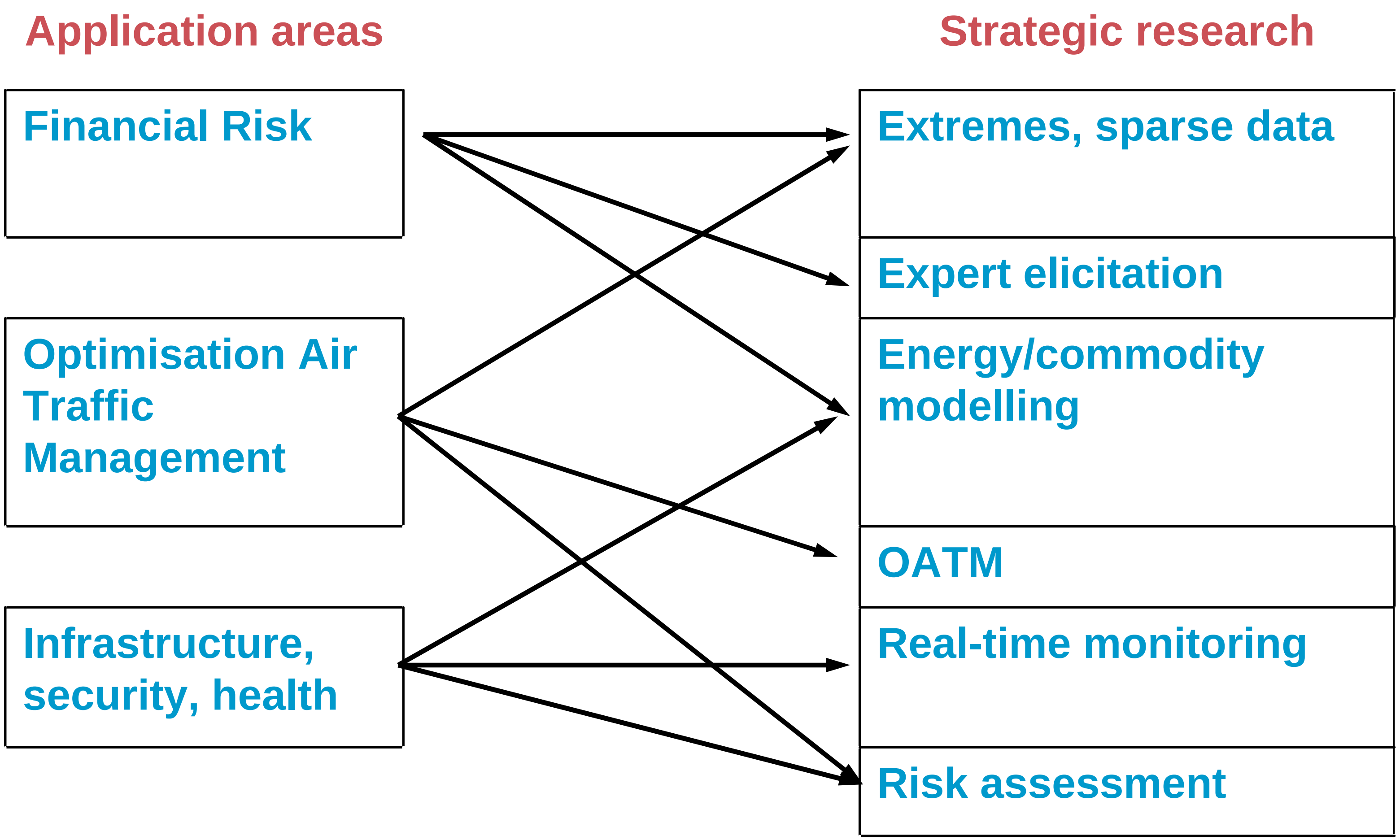
CSIRO Mathematical & Information Sciences

- **Commonwealth Scientific and Industrial Research Organization of Australia (CSIRO)**
  - National research agency formed in 1926.
  - Approx 6500 staff (Divisions: Industrial Physics, Minerals, Mathematical & Information Sciences, Marine and Atmospheric Research, etc.)
    www.csiro.au

- **Division of Mathematical and Information Sciences (CMIS)**
  - (over 100 researchers): Decision Technology, Biotechnology and Health Informatics, Environmental Informatics
    www.cmis.csiro.au

- **Quantitative Risk Management (QRM) group**
  - (approx. 20 staff): financial risk, infrastructure, environment risk, security, air-transport. Activities/modes of engagement: research, consulting, model development/validation, software development,… .
    www.cmis.csiro.au/QRM

CSIRO Quantitative Risk Management



Solutions

- Development of mathematical models and customised software according to the client methodology and specific needs.
- Assisting with development of new models and their implementation into software.
- Independent review and advice on risk models, methodology and software solutions.
- Independent validation of derivatives and risk measurement models.
- Time-series analysis of data.

Modes of engagement

There are many ways CSIRO can work with you to understand and quantify financial risks:

- Consultancy engagement.
- Contract research engagement.
- Collaborative projects.
- Software development.

Track Records in Financial Risk

- **<u>Derivative pricing:</u>** work on CSIRO option pricing software Reditus since 1999, consulting projects in 2005, 2006, 2007.
- **<u>FX option pricing:</u>** plug-ins for Fenics launched in 2005 – 100 users in overseas banks.
- **<u>Operational Risk:</u>** validation projects in 2000, model development projects in 2001, 2003, R&D/software projects 2004-2007.
- **<u>Market Risk:</u>** validation projects in 2004, 2005, model development consulting project in 2007.
- **<u>Credit Risk:</u>** validation projects in 1999, 2002; validation and model development projects 2004-ongoing.
- **<u>Underwriting risk:</u>** consulting projects in 1999, current proposals.
- **<u>Forecasting electricity/commodities:</u>** consulting projects in 2002, 2005, 2005-06, current proposals.
- **<u>Portfolio Management:</u>** model development projects in 2007, current proposals.
- **<u>Water/Carbon Trading:</u>** current proposals.
- **<u>Collaborators:</u>** Monash Uni, Cambridge Uni, ETH Zurich, UNSW, UTS, Macquarie Uni, Statistical Research Associates NZ
- **Industry clients**: CBA, ANZ, NAB, St George, Integral Energy, IAG, Fenics FX, Edgecap, Moore Capital, Ester Bank, Credit Swiss,… .

Financial Risk Management	Links	Other Risk Areas
• Market Risk • Credit Risk • Operational Risk • Underwriting risk • Derivative pricing • Interest Rates • Trading strategies • Portfolio Management • Commodity/Energy • Carbon/water trading • Model risk • Liquidity risk	•Extreme Value modelling •Dynamic control •Expert Elicitation •Bayesian methods •Dependence Modelling •Model validation •Computational methods (PDE, MCMC, Monte Carlo methods) •Time series analysis •High performance computing	•Air transport •Ecology/Environmental •Infrastructure •Security •Weather/Climate •Health

Extreme Value Analysis/Dependence

- **Air-transport (deviations from taxi centerline)**
- **Weather (rainfall, wind speed)**
- **Market Risk (tail of portfolio return distribution, derivatives)**
- **Credit/Operational Risk (tail of annual loss distribution)**
- **Electricity pricing (price spikes)**

Extreme Value Models

- **Block maxima – models for largest observations,** $M_n = \max(X_1, ..., X_n)$

$X_1, X_2, ...$ are iid **e.g. daily data grouped into quarterly blocks**

Limiting distribution: Generalized Extreme Value Distribution (GEV)

$$H(x) = \begin{cases} \exp[-(1+\xi \times x/\beta)^{-1/\xi}], \xi \neq 0 \\ \exp[-\exp(-x/\beta)], \quad \xi = 0 \end{cases}$$

- **Threshold exceedances – models for large observations exceeding some high level, *L*,** $Y_i = X_i - L$

e.g. operational losses exceeding 1mln

Limiting distribution: Generalized Pareto Distribution (GPD)

$$H(x) = \begin{Bmatrix} 1-(1+\xi x/\beta)^{-1/\xi}; & \xi \neq 0 \\ 1-\exp[-x/\beta]; & \xi = 0 \end{Bmatrix}$$

Market Risk

$$X_t = -(\ln S_t - \ln S_{t-1}) \approx (S_{t-1} - S_t)/S_{t-1}$$

$$X_t = \mu_t + \sigma_t Z_t$$

assume $Z_t, Z_{t-1}, \ldots$ are iid and X_t is stationary

For example: $\mu_t = \lambda X_{t-1}, \sigma_t^2 = \alpha_0 + \alpha_1 (X_{t-1} - \mu_{t-1})^2 + \beta \sigma_{t-1}^2$

$$VaR_q^t = \mu_{t+1} + \sigma_{t+1} VaR(Z)_q ; CVaR_q^t = \mu_{t+1} + \sigma_{t+1} CVaR(Z)_q$$

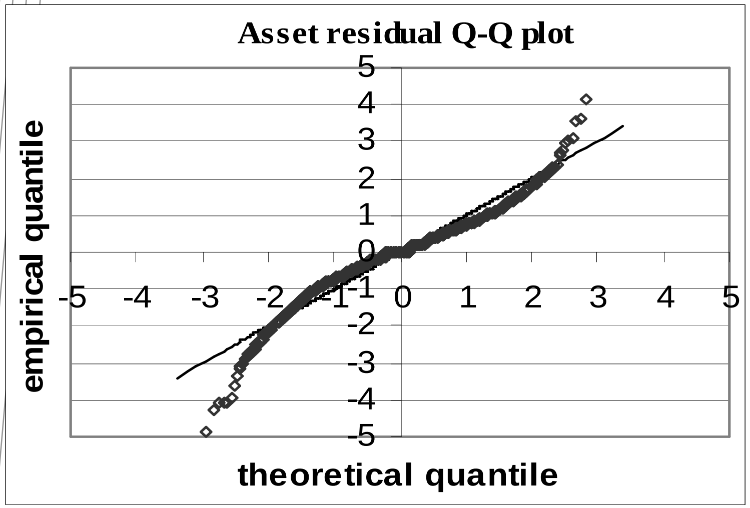
Z is modelled by GPD

Market Risk Regulatory Capital

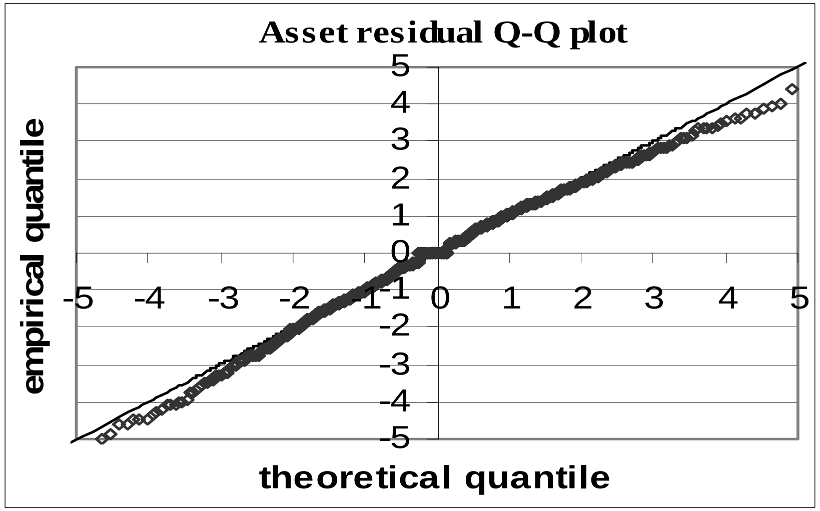
$$C(t) = \max\{\mathrm{VaR}_{0.99}^{t,10}, \frac{k}{60} \sum_{i=1}^{60} VaR_{0.99}^{t-i+1,10}\}$$

Example: API returns

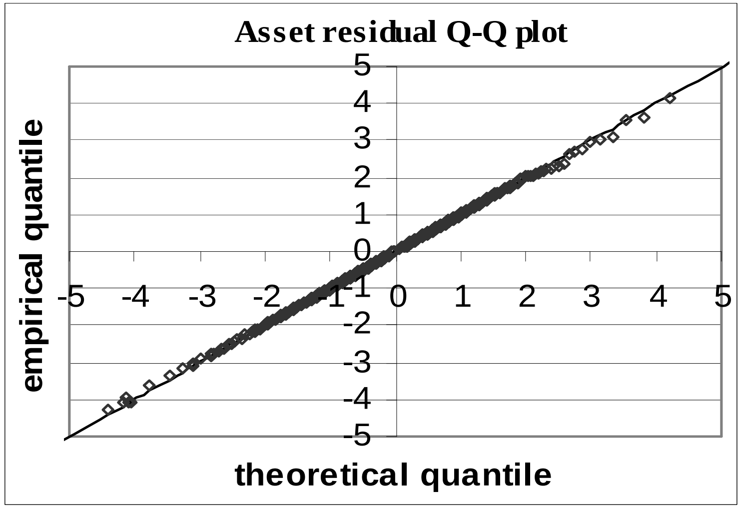
GARCH-Normal



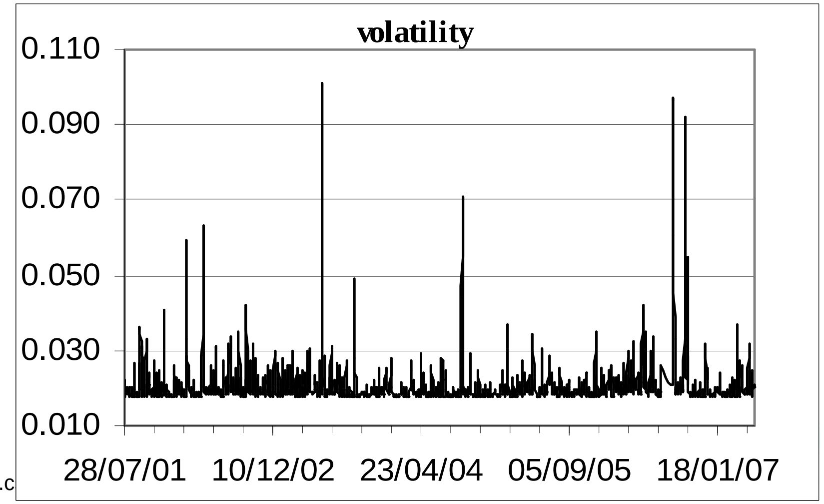
GARCH-Student t



GARCH-Generalized Pareto



GARCH-volatility



Derivative Pricing

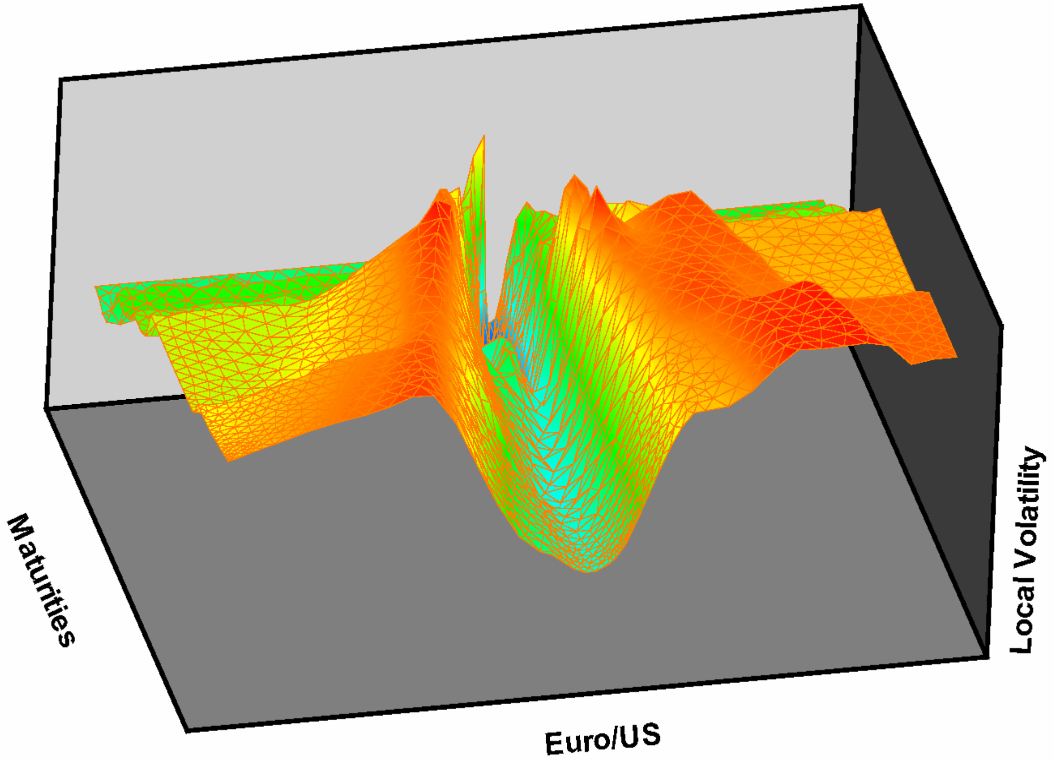
Option pricing : $Q_t = E[Payoff(S_T) | S_t]$

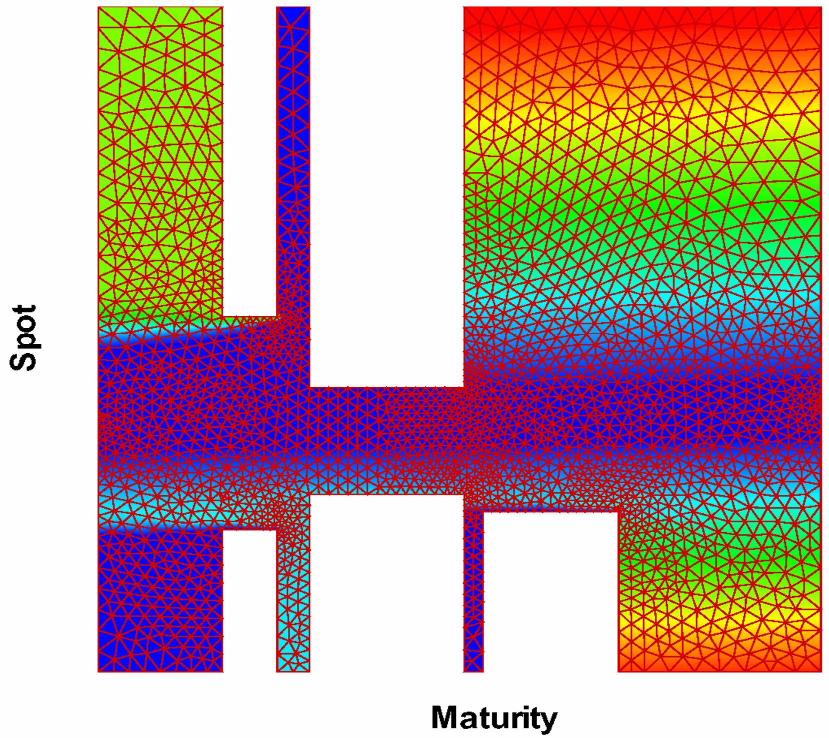
Modelling volatility skew :

e.g. local volatility models - $dS_t / S_t = (r_t - q_t)dt + \sigma(S_t, t)dW_t$,

stochastic volatility models, jump diffusion

Numerical Methods : Finite Element, Finite Difference, Monte Carlo methods.





Dependence modelling

Dependence modelling via Copula method

Consider rvs $X_1,...,X_d$ with $F_i(x_i)=\text{Prob}(X_i \le x_i)$

$$F_i^{-1}(U_i) \sim F_i \Rightarrow X_i = F_i^{-1}(U_i),\; U_i \sim U(0,1),\; F_i(X_i)=U_i \sim U(0,1)$$

$F(x_1,x_2,...,x_d)=\text{Prob}[X_1 \le x_1, X_2 \le x_2,...,X_d \le x_d]$ is joint cdf

$$F(x_1,x_2,...,x_d)$$

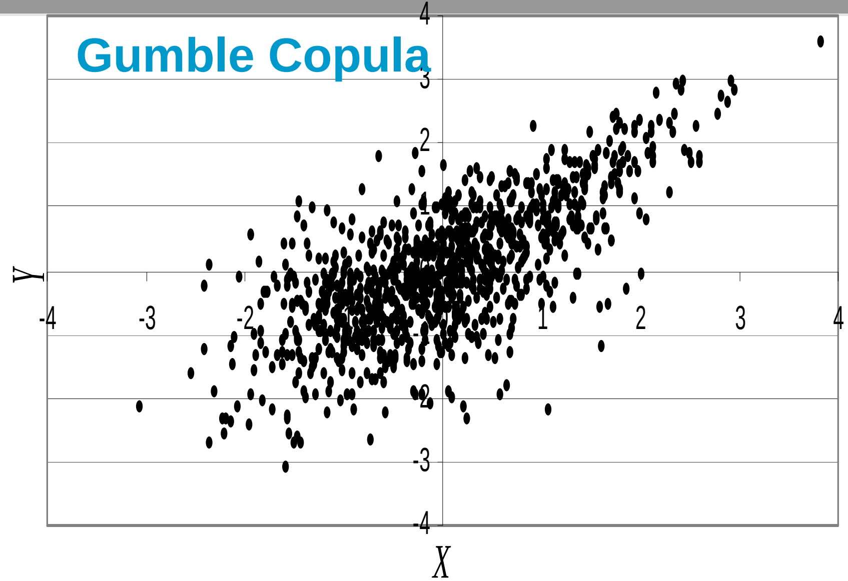
marginal behavior, F_i — copula $C(u_1,..,u_d)$

Copula is multivariate joint distribution of uniform random variables

$$C(u_1,u_2,...,u_d)=F(F_1^{-1}(u_1),F_2^{-1}(u_2),...,F_d^{-1}(u_d))$$

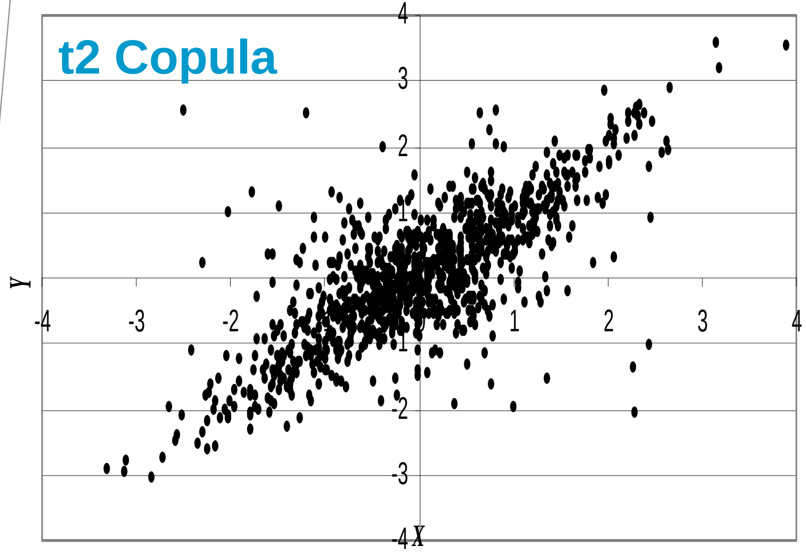
$$C(F_1(x_1),F_2(x_2),...,F_d(x_d))=F(x_1,x_2,...,x_d)$$

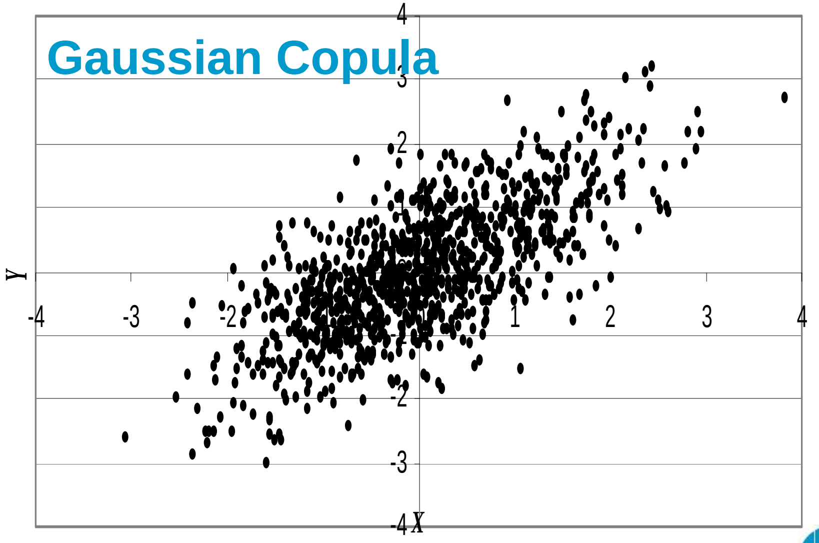
Dependence modelling via copula



$$X \sim Normal(0,1); Y \sim Normal(0,1)$$

$$corr(X,Y) = 0.7$$





Expert Elicitation/Bayesian methods

(combining internal & external data with expert opinion)

- **Ecology (estimation of fish density from gillnet catches)**
- **Operational Risk (estimation of loss frequency and severity)**
- **Air-transport (individual&collective risk of air-collisions)**
- **Insurance (pricing of policy premium)**
- **Markov Chain Monte Carlo methods (signal processing)**

- **Recent Publications:**
- **D. D. Lambrigger (ETH), P.V. Shevchenko (CSIRO) and M. V. Wüthrich (ETH),** 2007. *The Quantification of Operational Risk using Internal Data, Relevant External Data and Expert Opinions*. The Journal of Operational Risk.
- **Hans Bühlmann (ETH), P.V. Shevchenko (CSIRO) and M. Wüthrich (ETH),** 2006. A *"Toy" Model for Operational Risk Quantification using Credibility Theory.* The Journal of Operational Risk 2(1).
- **P.V. Shevchenko (CSIRO) and M. Wüthrich (ETH),** 2006. *Structural Modelling of Operational Risk using Bayesian Inference: combining loss data with expert opinions.* The Journal of Operational Risk 1(3), pp.3-26.

Combining internal data, industry data and expert opinions

Bayesian inference

observations $\mathbf{X} = (X_1, X_2, \ldots, X_n)$

parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_K)$

$$h(\mathbf{X}, \boldsymbol{\theta}) = h(\mathbf{X} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta}) = \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X})h(\mathbf{X})$$

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}) \propto h(\mathbf{X} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

$\pi(\boldsymbol{\theta})$ **prior distribution is estimated by expert/industry data**

$h(\mathbf{X} \mid \boldsymbol{\theta})$ **likelihood of internal observations**

$$\varphi(X_{n+1} \mid \mathbf{X}) = \int g(X_{n+1} \mid \boldsymbol{\theta}) \times \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}) d\boldsymbol{\theta}$$ **predictive distribution**

Combining internal data, external data and expert opinions

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}, \boldsymbol{\upsilon}) \propto h_1(\mathbf{X} \mid \boldsymbol{\theta})h_2(\boldsymbol{\upsilon} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

$\pi(\boldsymbol{\theta})$ **prior distribution is estimated by industry data**

$h_1(\mathbf{X} \mid \boldsymbol{\theta})$ **likelihood of internal observations**

$h_2(\boldsymbol{\upsilon} \mid \boldsymbol{\theta})$ **likelihood of expert opinions**

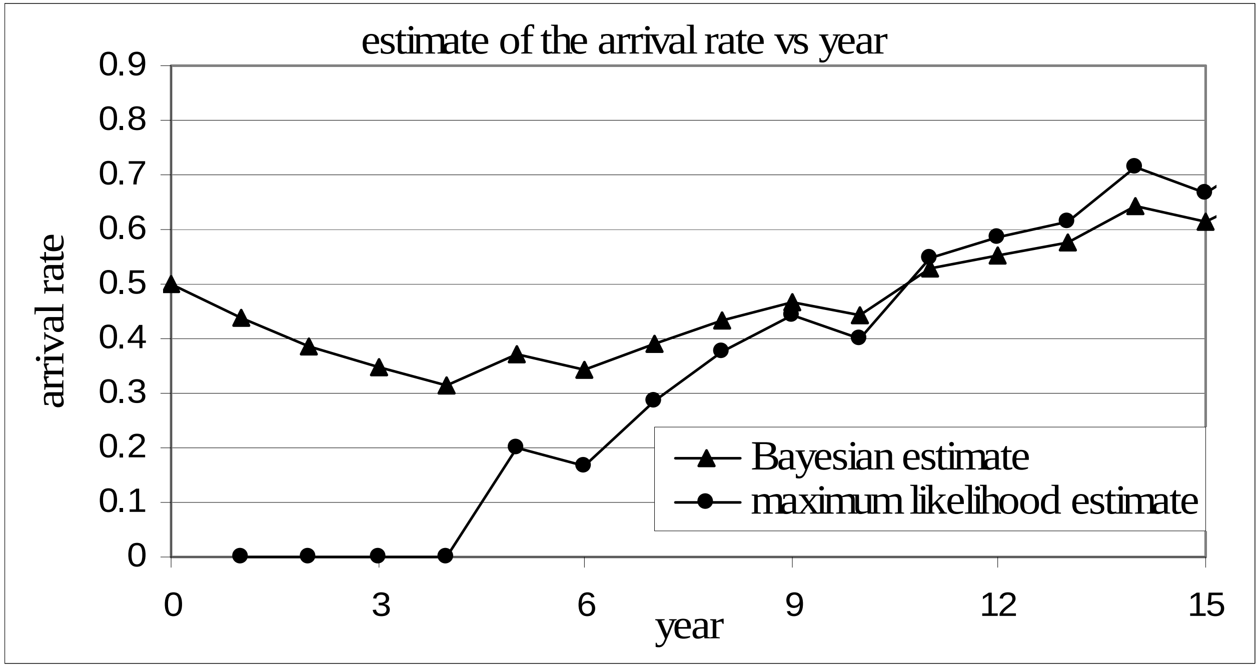
Example: combining expert opinion and internal data

Annual counts **N**=(0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 2, 1, 1, 2, 0) from Poisson $\lambda = 0.6$

expert opinions $E[\lambda] = 0.5, \Pr[0.25 \le \lambda \le 0.75] = 2/3 \Rightarrow \alpha \approx 3.41, \beta \approx 0.15$

$\hat{\lambda}_k = \hat{\alpha}_k \times \hat{\beta}_k$ the Bayesian estimator with Gamma prior $\alpha \approx 3.41, \beta \approx 0.15$

$\tilde{\lambda}_k = \frac{1}{k}\sum_{i=1}^{k} N_i$ the Maximum Likelihood estimator



Parameter Risk (uncertainty of parameters)

$$Z = \sum_{i=1}^{N} X_i - \text{annual loss;} \quad \mathbf{Y} = (\mathbf{X}, \mathbf{N}) - \text{past} \quad \text{observations}$$

$$\varphi(Z_{t+1} \mid \mathbf{Y}) = \int g(Z_{t+1} \mid \boldsymbol{\theta}) \times \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) d\boldsymbol{\theta} - \text{predictive distribution}$$

$$\hat{Q}_{0.999}^{B} - 0.999 \text{ quantile of} \quad \varphi(Z_{t+1} \mid \mathbf{Y})$$

$$\hat{Q}_{0.999} - 0.999 \text{ quantile of} \quad g(Z_{t+1} \mid \hat{\boldsymbol{\theta}})$$

$\hat{\boldsymbol{\theta}}$ is point estimator, e.g. maximum likelihood estimator

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) \propto h(\mathbf{Y} \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta}), \quad \text{Markov Chain Monte Carlo methods}$$

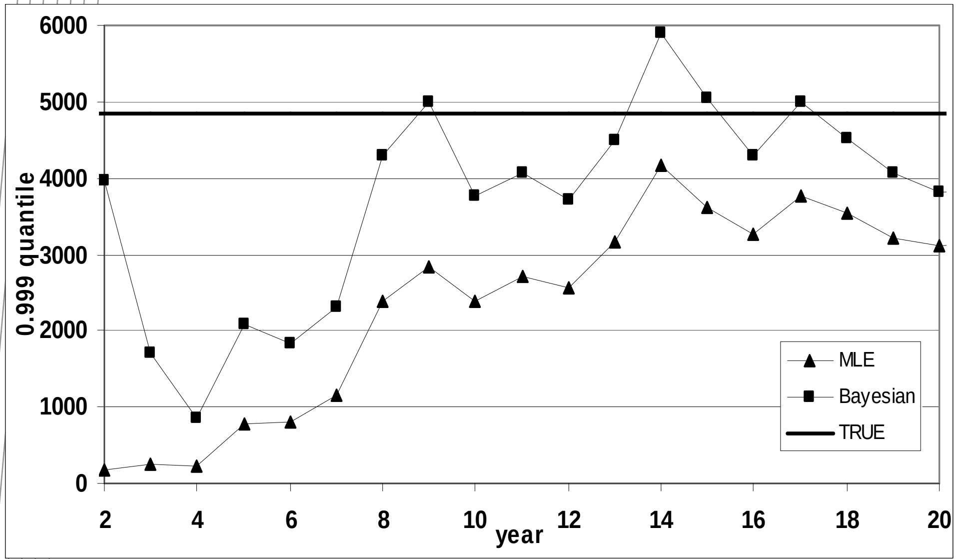
$$bias = E[\hat{Q}_{0.999}^{B} - \hat{Q}_{0.999}]$$

Normal approximation : $\hat{\pi}(\hat{\boldsymbol{\theta}} \mid \mathbf{Y})$ is $Normal(mean = \hat{\boldsymbol{\theta}}, \text{cov} = -\mathbf{I}^{-1})$

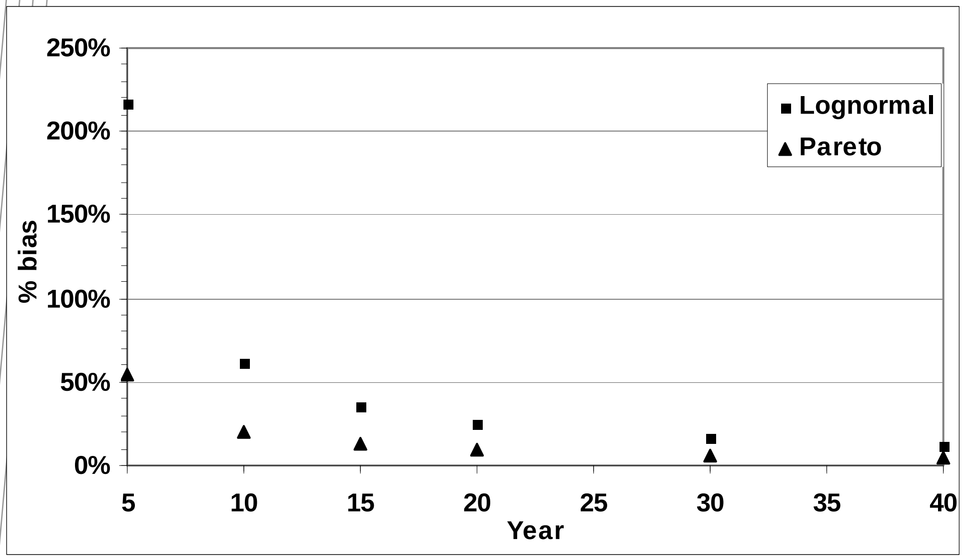
$$\ln \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) \approx \ln \hat{\pi}(\hat{\boldsymbol{\theta}} \mid \mathbf{Y}) + \frac{1}{2} \sum_{i,j} \mathbf{I}_{ij} (\theta_i - \hat{\theta}_i)(\theta_j - \hat{\theta}_j); \quad \mathbf{I}_{ij} = \left. \frac{\partial^2 \ln \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y})}{\partial \theta_i \partial \theta_j} \right|_{\theta = \hat{\theta}}$$

P.V. Shevchenko (2007). *Estimation of operational Risk Capital Charge under Parameter Uncertainty,* submitted to the Risk Magazine.

Parameter Risk (uncertainty of parameters)



Comparison of the 0.999 quantile estimators. Parameter uncertainty is ignored by (MLE) but is taken into account by (Bayesian).
MLE - maximum likelihood estimator
Bayesian - quantile of predictive distribution
Losses were simulated from *Poisson*(10) and *LN*(1,2).
Non-informative constant prior were used.



Relative bias in the 0.999 quantile estimator induced by the parameter uncertainty vs number of observation years.
(Lognormal) - losses were simulated from *Poisson*(10) and *LN*(1,2).
(Pareto) – losses were simulated from *Poisson*(10) and *Pareto*(2) with *L*=1.

Kalman/Particle filter techniques
(state-space models)

- **On-line monitoring of water quality**
- **Health surveillance**
- **Modelling commodities/interest rates**

State-space models: Kalman Filter

- **Measurement Equation** $\vec{F}_t = \vec{A} + \hat{B} \times \vec{X}_t + \vec{e}$
- **Transition Equation** $\vec{X}_{t+1} = \vec{M} + \hat{T} \times \vec{X}_t + \vec{\varepsilon}$

Commodity spot models:e.g. 2-factor convenience yield

S_t – spot price; δ_t – convenience yield, $F_{t,T}$ – futures prices

$$d \ln S_t = [\mu - \delta_t - \tfrac{1}{2}\sigma_1^2]dt + \sigma_1 dW_t^{(1)}$$

$$d\delta_t = [\alpha - \kappa_1 \delta_t - \kappa_2 X_t]dt + \sigma_2 dW_t^{(2)}; \quad E[dW_t^{(1)} dW_t^{(2)}] = \rho dt$$

$$F_{t,T} = E[S_T \mid S_t, \delta_t] \Rightarrow \ln F_{t,T} = A(T-t) + B(T-t)\ln S_t + C(T-t)\delta_t$$

Interest rate spot models:e.g. Vasicek model

r_t – short-term rate; $P_{t,T}$ – bond price

$$dr_t = \alpha[\gamma - r_t]dt + \sigma dW_t$$

$$P_{t,T} = A(t,T)\exp[-B(t,T)r_t] \Rightarrow \ln P_{t,T} = \ln A(T-t) - B(T-t)r_t$$

Emerging over-arching research topics

- Mixing internal & external data with expert opinions (credibility theory, Bayesian techniques)
- Dependence between risks: copula methods, structural models
- Compound point processes
- Modelling distribution tail: EVT, mixed distributions, splices
- Efficient Markov Chain Monte Carlo, Monte Carlo, finite element/finite difference methods
- State-space models (sequential Monte Carlo, Kalman filter)
- Time series analysis via chaos theory methods
- Modelling truncated/censored data
- Nonlinear optimization with constraints

CSIRO Mathematical & Information Sciences
Pavel Shevchenko

Phone: 02 9325 3218
Email: Pavel.Shevchenko@csiro.au
www.cmis.csiro.au/Pavel.Shevchenko

www.csiro.au

Thank you

CSIRO