A STUDY OF OPENINGS AND CLOSINGS WITH RECONSTRUCTION CRITERIA

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Abstract In this paper, a class of extended lower and upper levelings is investigated. Here, some conditions are imposed to the criteria for building extended lower and upper levelings in order to generate openings and closings with reconstruction criteria. The idea of building these new openings and closings comes from the notions of morphological filters by reconstruction and levelings. The main goal in studying these transformations consists in eliminating some inconveniences of the morphological opening (closing) and the opening (closing) by reconstruction, and to show the interest in image segmentation. In fact, while the morphological opening (closing) modifies the remaining structures, the opening (closing) by reconstruction sometimes reconstructs undesirable regions.

Keywords: Levelings, opening and closing by reconstruction, opening and closing with reconstruction criteria.

1. Introduction

Between the different morphological filters, the filters by reconstruction are powerful tools that enable us to eliminate undesirable features without affecting desirable ones. These transformations by reconstruction, that form a class of connected filters, have been studied and characterized in the binary case (Serra and Salambier [1], Crespo et al. [2], Serra [3] among others). Efficient algorithms for building filters by reconstruction were proposed by Vincent [4]. Due to the interesting characteristics of filters by reconstruction, an approach for characterizing connected filters was presented by Meyer [5]. By defining monotone planings and flattenings and by combining both notions, the levelings have been introduced. In the present work, the concept of levelings is used

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to build openings (closings) that enable us to avoid the inconveniences of the morphological opening (closing) and the opening (closing) by reconstruction. While the morphological opening modifies the remaining structures, the opening by reconstruction sometimes reconstructs undesirable regions. This paper is organized as follows. In section 2, the concepts of filters by reconstruction and levelings are presented. In section 3, we modify some criteria for constructing levelings in order to propose openings and closings with reconstruction criteria. In section 4, some properties of these transformations are described.

2. Some Basic Concepts of Morphological Filtering

2.1 Basic Notions of Morphological Filtering

The basic morphological filters are the morphological opening $\gamma_{\mu B}$ and the morphological closing $\varphi_{\mu B}$ with a given structuring element ; where, in this work, B is an elementary structuring element (3x3 pixels) that contains its origin. $\check{B}$ is the transposed set ($\check{B} = \{-x : x \in B\}$) and μ is an homothetic parameter. The morphological opening and closing are given, respectively, by:

$$\gamma_{\mu B}(f) = \delta_{\mu \check{B}}(\varepsilon_{\mu B}(f)) \quad \text{and} \quad \varphi_{\mu B}(f) = \varepsilon_{\mu \check{B}}(\delta_{\mu B}(f)) \tag{1}$$

where $\varepsilon_{\mu B}$ and $\delta_{\mu B}$ are the operations of erosion and dilation, respectively. In the following, we will avoid the elementary structuring element B. The expressions γ_μ,$\gamma_{\mu B}$ are equivalent (i.e. $\gamma_\mu = \gamma_{\mu B}$). When the homothetic parameter is $\mu = 1$, the structuring element B will also be avoided (i.e. $\delta_B = \delta$). When $\mu = 0$, the structuring element is a set made up of one point (the origin).

Another class of filters is composed by the opening and closing by reconstruction. When filters by reconstruction are built, the basic geodesic transformations, the geodesic dilation and the geodesic erosion of size 1, are iterated until idempotence is reached. Where the geodesic dilation and the geodesic erosion of size one are given by $\delta^1_f(g) = f \wedge \delta(g)$ with $g \leq f$ and $\varepsilon^1_f(g) = f \vee \varepsilon(g)$ with $g \geq f$, respectively. When the function g is equal to the erosion or the dilation of the original function, we obtain the opening and the closing by reconstruction:

$$\widetilde{\gamma}_\mu(f) = \lim_{n\to\infty} \delta^n_f(\varepsilon_\mu(f)) \quad \widetilde{\varphi}_\mu(f) = \lim_{n\to\infty} \varepsilon^n_f(\delta_\mu(f)) \tag{2}$$

2.2 Connectivity and Levelings

The interesting filtering characteristics of filters by reconstruction, that form a class of connected filters, motivated the study of connected filters using the notion of connectivity class. Serra [6] introduced this notion in morphology starting from the following definition:

Definition 1 *A connectivity class C is defined on the subset of a set E when: 1. $\emptyset \in C$ and $\forall\ x \in E, \{x\} \in C$, 2. For each family $\{C_i\}$ of C, $\cap C_i \neq \emptyset \Rightarrow \cup C_i \in C$.*

This definition is equivalent to the connectivity characterized by openings.

Definition 2 *The definition of a connectivity class C is equivalent to the definition of a family of openings $\{\gamma_x, x \in E\}$ such that: 1. $\forall x \in E$, $\gamma_x(\{x\}) = \{x\}$, 2. $\forall x, y \in E$ and $X \subseteq E$, $\gamma_x(X)$ and $\gamma_y(X)$ are either equal or disjoint, 3. $\forall x \in E$ and $X \subseteq E$, $x \notin X \Rightarrow \gamma_x(X) = \emptyset$*

Another approach for characterizing connectivity was introduced by Meyer [5] using the notion of levelings. In the present work, we study the lower and upper levelings, defined below.

Criterion 1 *A function g is a lower-leveling (resp. an upper-leveling) of a function f if and only if $[g \geq f \wedge \delta(g)]$ (resp. $[g \leq f \vee \varepsilon(g)]$).*

Principally, we will focus on the criteria for building extended lower and upper levelings given by:

Criterion 2 *A function g is a lower-leveling (resp. an upper-leveling) of a function f if and only if $[g \geq f \wedge (g \vee \gamma_\lambda \delta(g))]$ (resp. $[g \leq f \vee (g \wedge \varphi_\lambda \varepsilon(g))]$)*

Criterion 3 *A function g is a lower-leveling (resp. an upper-leveling) of a function f if and only if $[g \geq f \wedge (g \vee \delta\gamma_\lambda(g))]$ (resp. $[g \leq f \vee (g \wedge \varepsilon\varphi_\lambda(g))]$)*

Criterion (2) has been used in Meyer and Maragos [7], while criterion (3) has been proposed in Meyer [5].

3. Openings and Closings with Reconstruction Criteria

As it was written before, unlike traditional opening and closing, opening and closing by reconstruction preserve shapes in the image , but some times, these connected transformations reconstruct too much. Then, the goal in this section is to introduce openings (resp. closings) that enable us to obtain intermediate results between the morphological opening (resp. closing) and the opening (resp. closing) by reconstruction. The process to build these transformations involves the use of a reference image and a marker image as in the reconstruction case. Thus, a reconstruction process of the marker inside the reference is made, as it is the case in the reconstruction transformations, but a reconstruction criterion is taken into account. We principally present the case of the openings, but a similar procedure can be made for the closings.

Let us analyze criterion (2). As for the opening by reconstruction, the relationship $f \wedge (g \vee \gamma_\lambda \delta(g))$ is iterated until stability (at each step $g = f \wedge (g \vee \gamma_\lambda \delta(g)))$, but this expression will be simplified by some condition imposed to the function g (similar for $f \vee (g \wedge \varphi_\lambda \varepsilon(g)))$. The function given by $g = \gamma_\mu(f)$ is selected as the marker image and not the erosion function as it is the case for the opening by reconstruction. However, an opening by reconstruction can be also obtained using the morphological opening as the marker image: $\widetilde{\gamma}_\mu(f) = \lim_{n\to\infty} \delta_f^n(\varepsilon_\mu(f)) = \lim_{n\to\infty} \delta_f^n(\gamma_\mu(f))$.

On the other hand, the morphological opening and the morphological dilation satisfy the following property (Serra [6]):

Property 1 *For all pairs of parameters λ_1, λ_2 with $\lambda_1 \leq \lambda_2$, $\delta_{\lambda_2}(g) = \gamma_{\lambda_1}(\delta_{\lambda_2}(g))$.*

By applying this property to $\gamma_\lambda\delta(g)$ using $g = \gamma_\mu(f)$ as a marker image, we get for all $\lambda \leq \mu + 1$:

$$\gamma_\lambda\delta(g) = \gamma_\lambda\delta(\gamma_\mu(f)) = \gamma_\lambda\delta_{\mu+1}\varepsilon_\mu(f) = \delta_{\mu+1}\varepsilon_\mu(f) = \delta\gamma_\mu(f)$$
$$\textit{and} \qquad \gamma_\mu(f) \leq \gamma_\lambda\delta(\gamma_\mu(f)) = \delta\gamma_\mu(f)$$

Thus, the term $g \vee \gamma_\lambda\delta(g)$ of $f \wedge (g \vee \gamma_\lambda\delta(g))$ is simplified to $\gamma_\lambda\delta(g)$ and we will work with the expressions: $\omega^1_{\lambda,f}(g) = f \wedge \gamma_\lambda\delta(g)$ and $\alpha^1_{\lambda,f}(g) = f \vee \varphi_\lambda\varepsilon(g)$, with the conditions $g = \gamma_\mu(f)$ for $\omega^1_{\lambda,f}(g)$ and $g = \varphi_\mu(f)$ for $\alpha^1_{\lambda,f}(g)$. Specifically, the equation $\gamma_\lambda\delta(\gamma_\mu(f)) = \delta\gamma_\mu(f)$ expresses that when the marker image is given by $g = \gamma_\mu(f)$ for $\lambda \leq \mu + 1$, the output images of successive iterations of the operation $\gamma_\lambda\delta$ are similar to those generated by the successive iterations of δ. However, the reconstruction process changes when the reference image f is used. In this case, the reference image modifies the reconstruction process of successive iterations of $\gamma_\lambda\delta$, and the opening γ_λ takes the role of a reconstruction criterion by restricting the reconstruction to some regions of the reference image f. A dual transformation is obtained by using $\alpha^1_{\lambda,f}(g)$. When stability is reached, an opening and a closing of size μ are obtained. They are given, $\forall\ \lambda \leq \mu + 1$, by:

$$\gamma_{\lambda,\mu}(f) = \lim_{n\to\infty} \omega^n_{\lambda,f}(\gamma_\mu(f)) \qquad \varphi_{\lambda,\mu}(f) = \lim_{n\to\infty} \alpha^n_{\lambda,f}(\varphi_\mu(f)) \tag{3}$$

Now, let us compare criterion (3) using expression $f \wedge (g \vee \delta\gamma_\lambda(g))$, with the one used by criterion (2): $f \wedge (g \vee \gamma_\lambda\delta(g))$. It is possible to show that the output images of the successive iterations of $(g \vee \delta\gamma_\lambda(g))$ are similar to those of $\delta(g)$, using $g = \gamma_\mu(f)$ as the marker. Since $\forall f$, $\gamma_\mu\gamma_\lambda(f) = \gamma_\lambda\gamma_\mu(f) = \gamma_{max(\mu,\lambda)}(f)$, ($max(\mu, \lambda)$ is the maximum value between μ and λ): $\delta\gamma_\lambda(g) = \delta\gamma_\lambda(\gamma_\mu(f)) = \delta(\gamma_\mu(f)) \quad \forall\lambda \leq \mu$.

Thus, the output images at each iteration of expressions $\delta\gamma_\lambda$ and $\gamma_\lambda\delta$, applied to $g = \gamma_\mu(f)$, are similar with the conditions $\forall\lambda \leq \mu$ and $\forall\lambda \leq \mu + 1$, respectively. Also, the expression $f \wedge (g \vee \delta\gamma_\lambda(g))$ can be simplified by $f \wedge \delta\gamma_\lambda(g)$ with $g = \gamma_\mu(f)$. However, by iterating both expressions, $f \wedge \delta\gamma_\lambda(g)$ and $f \wedge \gamma_\lambda\delta(g)$ until stability, different output images are obtained. Then, other openings and closings of size μ can be established using the operations $\omega^1_{\lambda,f} = f \wedge \delta\gamma_\lambda$ and $\alpha^1_{\lambda,f} = f \vee \varepsilon\varphi_\lambda$, with the condition $\lambda \leq \mu$:

$$\widehat{\gamma}_{\lambda,\mu}(f) = \lim_{n\to\infty} \omega^n_{\lambda,f}(\gamma_\mu(f)) \qquad \widehat{\varphi}_{\lambda,\mu}(f) = \lim_{n\to\infty} \alpha^n_{\lambda,f}(\varphi_\mu(f)) \tag{4}$$

The interest of these transformations (closings) is illustrated in Fig. 1. This is the typical problem of the connected operators, reported in [8], called leakage. Using the morphological closing, the word "LETTER" (Fig.1(a)) has been eliminated but the remaining structures are modified (see Fig. 1(b)); while using the closing by reconstruction the remaining structures are not changed but the word has not been correctly eliminated (Fig.1(c)). Finally, by applying the closings with reconstruction criteria, $\varphi_{\lambda,\mu}$ and $\widehat{\varphi}_{\lambda,\mu}$, we observe in Figs. 1(d) and 1(e) that the reaming structures are not considerably modified by both closings, and the word is eliminated.

Figure 1. (a) Original image; (b) morphological closing with $\mu = 7$; (c) closing by reconstruction with $\mu = 7$; (d) and (e) closings with reconstruction criteria using $\varphi_{\lambda,\mu}$ and $\widehat{\varphi}_{\lambda,\mu}$, respectively, with $\mu = 7$ and $\lambda = 3$.

4. Some properties of the openings with reconstruction criteria

4.1 Criteria option for building the openings

Let us study the behavior of the opening $\gamma_{\lambda,\mu}$ (Eqn. (3)), in the binary case, using expression $X \cap \gamma_\lambda \delta(\gamma_\mu(X))$. We have at step k:

$$\omega^k_{\lambda,X}(\gamma_\mu(X)) = X \cap \gamma_\lambda \delta(\omega^{k-1}_{\lambda,X}(\gamma_\mu(X))) \qquad \forall \lambda \leq \mu + 1$$

In this case, the output image $\omega^k_{\lambda,X}(\gamma_\mu(X))$ is composed by the union of the set $\omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ and the points $x \in X$ ($x \notin \omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$) such that for each x there exits a point y with $\lambda B_y \subset \delta(\omega^{k-1}_{\lambda,X}(\gamma_\mu(X)))$ and $x \in \lambda B_y$. Figure (2) shows a simple example that illustrates the different steps of the transformation. Figures 2(a) and 2(b) show the original set X and the opening $\gamma_\mu(X)$ of size $\mu = 2$ in dark gray color. Figures 2(c) to 2(e) illustrate the reconstruction process, steps one, two, and three, respectively, with $\lambda = 1$. In Fig. 2(h) the structuring element λB_y at point y in dark gray color and its dilated set $\delta(\lambda B_y)$ in light gray color are illustrated. At step one, the input set $\omega^0_{\lambda,X}(\gamma_\mu(X)) = \gamma_\mu(X)$ is dilated, and then tested by the set λB. The points in light gray color (three points) in Fig. 2(c) hit the structuring element ($\lambda B_y \subset \delta(\gamma_\mu(X))$) and they are added to $\gamma_\mu(X)$ to obtain $\omega^1_{\lambda,X}(\gamma_\mu(X))$; while at step two, using the same procedure, only two points (Fig. 2(d)) are added to $\omega^1_{\lambda,X}(\gamma_\mu(X))$ to form $\omega^2_{\lambda,X}(\gamma_\mu(X))$. At step three in Fig. 2(e), the stability is reached by adding the points $x1, x2, x3$ to $\omega^2_{\lambda,X}(\gamma_\mu(X))$ to form $\omega^3_{\lambda,X}(\gamma_\mu(X))$.

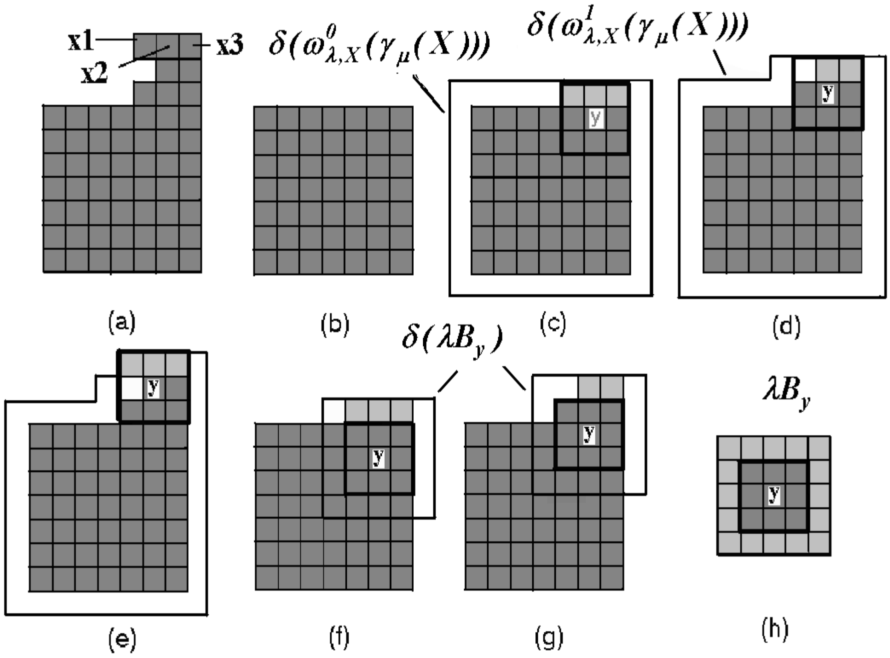

Figure 2. (a) Original image; (b) morphological opening $\mu = 2$,(c), (d) and (e) output sets of the operator $\omega^k_{\lambda,X}(\gamma_\mu(X))$,for k=1,2, and 3 respectively, using $X \cap \gamma_\lambda \delta(\gamma_\mu(X))$; (f) and (g) output sets of the operator $\omega^k_{\lambda,X}(\gamma_\mu(X))$,for k=1,2, respectively, using $X \cap \delta\gamma_\lambda(\gamma_\mu(X))$; (h) structuring element λB and its dilated $\delta(\lambda B)$.

Now, consider the opening $\widehat{\gamma}_{\lambda,\mu}$ in the binary case using the expression $X \cap \delta\gamma_\lambda(\gamma_\mu(X))$. At step k before stability, we have $\forall \lambda \leq \mu$:

$$\omega^k_{\lambda,X}(\gamma_\mu(X)) = X \cap \delta\gamma_\lambda(\omega^{k-1}_{\lambda,X}(\gamma_\mu(X)))$$

A point $x \in X$ with $x \notin \omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ belongs to $\omega^k_{\lambda,X}(\gamma_\mu(X))$, if there exists $\lambda B_y \subset \omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ such that $x \in \delta(\lambda B_y)$. Figures 2(f) and 2(g) illustrate the reconstruction of X using $\lambda = 1$. At step one, the input set $\omega^0_{\lambda,X}(\gamma_\mu(X)) = \gamma_\mu(X)$ is tested by λB and the points in light gray color (three points) in Fig. 2(f) that hit the dilated of the translated structuring element at point y ($\delta(\lambda B_y)$) are added to $\gamma_\mu(X)$ to obtain $\omega^1_{\lambda,X}(\gamma_\mu(X))$; while at step two, using the same procedure, only two points (light gray color in Fig. 2(g)) are added to $\omega^2_{\lambda,X}(\gamma_\mu(X))$. At step two, the stability is reached because it is not possible to find a point y with $B_y \subset \omega^2_{\lambda,X}(\gamma_\mu(X))$ such that at least one of the points x_1, x_2, x_3 belongs to $\delta(B_y)$.

Using this previous analysis, let us illustrate the differences between both openings. Figure 3(a) shows the original image, while Fig. 3(b) illustrates the binary image obtained by a threshold of the original image between 80 and 255 gray levels. The morphological opening and the opening by reconstruction, with

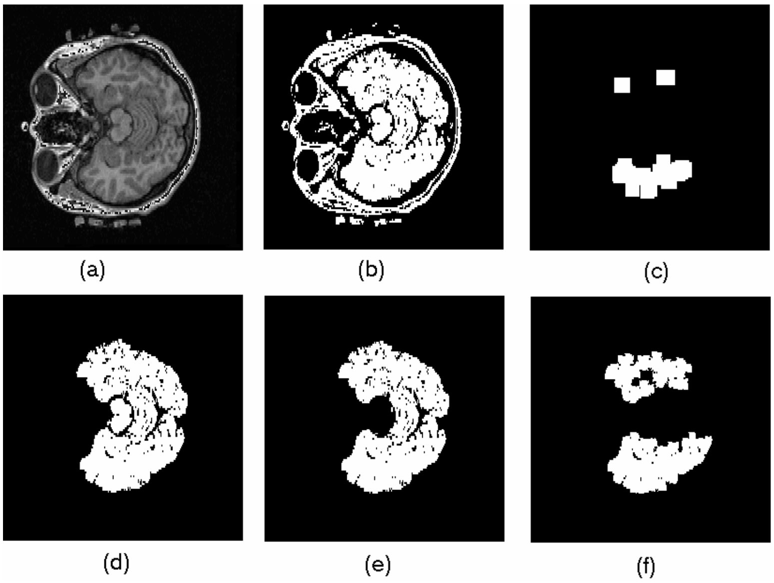

Figure 3. (a) Original image; (b) binary image X, (c) morphological opening with $\mu = 7$; (d) opening by reconstruction with $\mu = 7$; (e) and (f) openings with reconstruction criteria using $\gamma_{\lambda,\mu}$ and $\widehat{\gamma}_{\lambda,\mu}$, respectively, with $\mu = 7$ and $\lambda = 3$

$\mu = 7$, are illustrated in Figs. 3(c) and 3(d). Finally, the openings $\gamma_{\lambda,\mu}$ and $\widehat{\gamma}_{\lambda,\mu}$ with parameters $\mu = 7$ and $\lambda = 3$ were used to compute the output images in Figs. 3(e) and 3(f), respectively. Notice that when $\gamma_{\lambda,\mu}$ is used, the small holes do not affect considerably the reconstruction. Intuitively, this behavior is due to the conditions required to add a point $x \in X$ to the output image. At step k a point $x \in X$ such that $x \notin \omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ belongs to $x \in \omega^{k}_{\lambda,X}(\gamma_\mu(X))$, if there exists $\lambda B_y \subset \delta(\omega^{k-1}_{\lambda,X}(\gamma_\mu(X)))$ and $x \in \lambda B_y$. This means that at step k, the output image at step $k-1$ is dilated $\delta(\omega^{k-1}_{\lambda,X}(\gamma_\mu(X)))$, and some small holes are filled before they are tested by λB_y. Then, it seems that the opening $\gamma_{\lambda,\mu}$ has a better behavior than the opening $\widehat{\gamma}_{\lambda,\mu}$, but this depends on practical problems. For instance, in the example in Fig. 1, the closing $\widehat{\varphi}_{\lambda,\mu}(f)$ eliminates better the word "LETTER" than the closing $\varphi_{\lambda,\mu}(f)$. Furthermore, the opening $\widehat{\gamma}_{\lambda,\mu}$ and the closing $\widehat{\varphi}_{\lambda,\mu}$ have other properties that will be described below.

In particular, the extreme values of λ in the opening $\widehat{\gamma}_{\lambda,\mu}(f)$ $(\lambda = 0, \lambda = \mu)$ are well-defined. Observe that for $\lambda = 0$, the expression $\omega^1_{\lambda,f} = f \wedge \delta\gamma_\lambda(\gamma_\mu(f))$ becomes $\omega^1_{0,f} = f \wedge \delta(\gamma_\mu(f))$ which is the geodesic dilation of size 1. Thus, $\widehat{\gamma}_{0,\mu}(f) = \lim_{n\to\infty} \omega^n_{0,f}(\gamma_\mu(f)) = \lim_{n\to\infty} \delta^n_f(\gamma_\mu(f)) = \widetilde{\gamma}_\mu(f)$. For $\lambda = \mu$, at the first iteration we have $\omega^1_{\mu,f}(\gamma_\mu(f)) = f \wedge \delta\gamma_\mu(\gamma_\mu(f)) = f \wedge \delta\gamma_\mu(f)$, then, $\gamma_\mu(f) \le \omega^1_{\mu,f}(\gamma_\mu(f)) = f \wedge \delta\gamma_\mu(f) = \delta^1_f\gamma_\mu(f) \le f$. At the second step, $\omega^2_{\mu,f}(\gamma_\mu(f)) = f \wedge \delta\gamma_\mu(\delta^1_f\gamma_\mu(f))$ and, since γ_μ is a strong filter we obtain $\omega^2_{\mu,f}(\gamma_\mu(f)) = f \wedge \delta\gamma_\mu(f) = \delta^1_f\gamma_\mu(f)$. Thus, for $\lambda = \mu$, $\widehat{\gamma}_{\mu,\mu}(f) = \delta^1_f\gamma_\mu(f)$, and we have the following relation between the openings γ_μ, $\widehat{\gamma}_{\lambda,\mu}$ and $\widetilde{\gamma}_\mu$: $\forall f$,

$\gamma_\mu(f) \leq \widehat{\gamma}_{\lambda,\mu}(f) \leq \widetilde{\gamma}_\mu(f)$ with $\lambda \leq \mu$. This is not the case for the opening $\gamma_{\lambda,\mu}$. For $\lambda = 0$ we have that $\widehat{\gamma}_{0,\mu}(f) = \widetilde{\gamma}_\mu(f)$, but also for $\lambda = 1$. For $\lambda = 1$, the output image at step k is given by $\omega^k_{1,f}(\gamma_\mu(f)) = f \wedge \gamma\delta(\omega^{k-1}_{1,f}(\gamma_\mu(f)))$. Using property (1), $\omega^k_{1,f}(\gamma_\mu(f)) = f \wedge \delta(\omega^{k-1}_{1,f}(\gamma_\mu(f)))$, which is the geodesic dilation of size 1. However, as for the opening $\widehat{\gamma}_{\lambda,\mu}$, there exits an inclusion relation between γ_μ, $\gamma_{\lambda,\mu}$ and $\widetilde{\gamma}_\mu$: $\forall f$, $\gamma_\mu(f) \leq \gamma_{\lambda,\mu}(f) \leq \widetilde{\gamma}_\mu(f)$ with $\lambda \leq \mu + 1$.

4.2 Some comments about connectivity

The goal in binary image segmentation is to split the connected components into a set of elementary shapes. In [8], the segmentation by openings using the connectivity opening proposed by Ronse [9] has been reported.

Segmentation by openings. Given a family of connected point-wise openings γ_x, and an opening γ_μ, a new family of connected point-wise openings, σ_x can be created by the following rule: $\sigma_x(X) = \gamma_x\gamma_\mu(X)$ if $x \in \gamma_\mu(X)$, $\sigma_x(X) = x$ if $x \in X/\gamma_\mu(X)$, and $\sigma_x(X) = \emptyset$ if $x \notin X$.

The opening γ_μ must verify the property: $\gamma_\mu\gamma_x\gamma_\mu = \gamma_x\gamma_\mu$. In other words, every connected component of an invariant of γ_μ is itself an invariant of γ_μ.

The main problem of this approach is that in practice this way of segmenting the connected components leads to a loss of the shape information of the remaining structures. This drawback can be attenuated using the openings with reconstruction criteria as illustrated by Fig. 1, in particular, using the opening defined by equation (4) since the opening given by equation (3) does not satisfy the condition to create this connected point-wise opening. In fact, during the reconstruction process for obtaining the opening $\gamma_{\lambda,\mu}$ the input set is dilated at each iteration before it is tested by λB, and the reconstruction process of two or more connected components can interact to obtain the output image. This is not the case for the opening $\widehat{\gamma}_{\lambda,\mu}$ since at each step the input set is tested by λB before the structuring element is dilated. Let us illustrate this behavior by analyzing the operator $\omega^k_{\lambda,X}$ when it is iterated to build $\widehat{\gamma}_{\lambda,\mu}$.

A set X is arc-wise connected if any pair of points x, y of X is linked by a path, entirely included in X. Where a path between two pixels of cardinal m is an m-tuple of pixels $x_0, x_1, ..., x_m$ such that $x = x_0$ and $x_m = y$ with x_k, x_{k+1} neighbors for all k. Similar, a path to characterize the connected components of $\widehat{\gamma}_{\lambda,\mu}(X)$ can be defined under some conditions. It is interesting to observe that the translates of λB, inside $\omega^{k-1}_{\lambda,X}(\gamma_\mu(X)$, that hit the boundaries of the set $\omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ enable us to decide which points at a distance 1 are added to the set $\omega^{k-1}_{\lambda,X}(\gamma_\mu(X))$ to form the set $\omega^k_{\lambda,X}(\gamma_\mu(X))$. Therefore, a point $y \in X$ with $y \notin \gamma_\mu(X)$ is achieved by the reconstruction process coming from $\gamma_\mu(X)$ at step m, if there exists a chain of points $x_0, x_1, ..., x_m$, with x_k, x_{k+1} neighbors, such that $\forall k$, $\lambda B_{x_k} \subset \omega^k_{\lambda,\mu}(\gamma_\mu(X))$ ($\omega^0_{\lambda,\mu} = \gamma_\mu(X)$) and λB_{x_k} hits the boundaries of $\omega^k_{\lambda,\mu}(\gamma_\mu(X))$, and $y \in \delta(\lambda B_{x_m})$. Observe that not only the size of the structuring element λ plays a fundamental role in the reconstruction process, but also the marker $\gamma_\mu(X)$ as illustrated by the following example in Fig. 4. The goal is to extract the leaf (tomato plant leaf) in the original image in Fig.

4(a). After a binarization step of the original image in Fig. 4(a), we obtain a set X formed by the leaf and other regions that are arc-wise connected to the leaf (Fig. 4(b)). The morphological openings $\gamma_{\mu 1}(X)$ and $\gamma_{\mu 2}(X)$ with $\mu 1 = 82$ and $\mu 2 = 155$ are shown in gray color in Figs. 4(c) and 4(d), respectively, while Fig 4(e) illustrates the output image of the openings $\widehat{\gamma}_{\lambda,\mu 1}$ and $\widehat{\gamma}_{\lambda,\mu 2}$ with $\lambda = 18$ ($\widehat{\gamma}_{\lambda,\mu 1}(X) = \widehat{\gamma}_{\lambda,\mu 2}(X)$). This is not the case of the output image in Fig. 4(f) computed by $\widehat{\gamma}_{\lambda,\mu}$ using $\mu = 81$ and $\lambda = 18$. In this example ($\lambda = 18$), the same output image is obtained by $\widehat{\gamma}_{\lambda,\mu}(X)$ for μ between $\mu 1 = 82$ and $\mu 2 = 155$, where the value $\mu 2 = 155$ is the critical element of the morphological opening, i.e. $\gamma_{\mu 2}(X) \neq \emptyset$ and $\gamma_{\mu 2+1}(X) = \emptyset$. Thus, the well-known property of the opening by reconstruction, expressing that for a given connected component X, such that $\gamma_x \widetilde{\gamma}_{\mu 1}(X) \neq \emptyset$ and $\gamma_x \widetilde{\gamma}_{\mu 2}(X) \neq \emptyset \Rightarrow \gamma_x \widetilde{\gamma}_{\mu 1}(X) = \gamma_x \widetilde{\gamma}_{\mu 2}(X)$ is verified by the opening $\widehat{\gamma}_{\lambda,\mu}$ in this example for all $\mu 1$, $\mu 2$ between 82 and 155, ($\gamma_x \widehat{\gamma}_{\lambda,\mu 1}(X) \neq \emptyset$ and $\gamma_x \widehat{\gamma}_{\lambda,\mu 2}(X) \neq \emptyset \Rightarrow \gamma_x \widehat{\gamma}_{\lambda,\mu 1}(X) = \gamma_x \widehat{\gamma}_{\lambda,\mu 2}(X)$).

In the general case, we need some conditions imposed to the input set as expressed by the following property.

Property 2 *Let A be an arc-wise connected component such that $\forall x, y \in A$ there exists a path $x_0, ..., x_m$, with x_k, x_{k+1} neighbors $\forall k$, entirely included in A, with $\lambda B_{x_k} \subset A$ $\forall k$ and such that $x \in \delta(\lambda B_{x_0})$ and $y \in \delta(\lambda B_{x_m})$. Then, $\forall \mu_1, \mu_2$ with $\lambda \leq \mu_1 \leq \mu_2$ such that $\widehat{\gamma}_{\lambda,\mu_1}(A) \neq \emptyset$ and $\widehat{\gamma}_{\lambda,\mu_2}(A) \neq \emptyset \Rightarrow \widehat{\gamma}_{\lambda,\mu_1}(A) = \widehat{\gamma}_{\lambda,\mu_2}(A) = A$*

5. Conclusion

In the present work, a study of a class of extended lower an upper levelings has been made in order to build openings and closings with reconstruction criteria. These openings (resp. closings) with reconstruction criteria enable us to obtain intermediate results between the morphological opening (resp. closing) and the opening (resp. closing) by reconstruction. The main property of these transformations is that they do not reconstruct those regions linked by thin connections by preserving the shape of the remaining structures.

Acknowledgments

We would like to thank Marcela Sanchez Alvarez for her careful revision of the English version. The author I. Terol thanks Diego R. and Darío T.G. for their great encouragement. This work was partially funded by the government agency CONACyT (Mexico) under the grant 25641-A.

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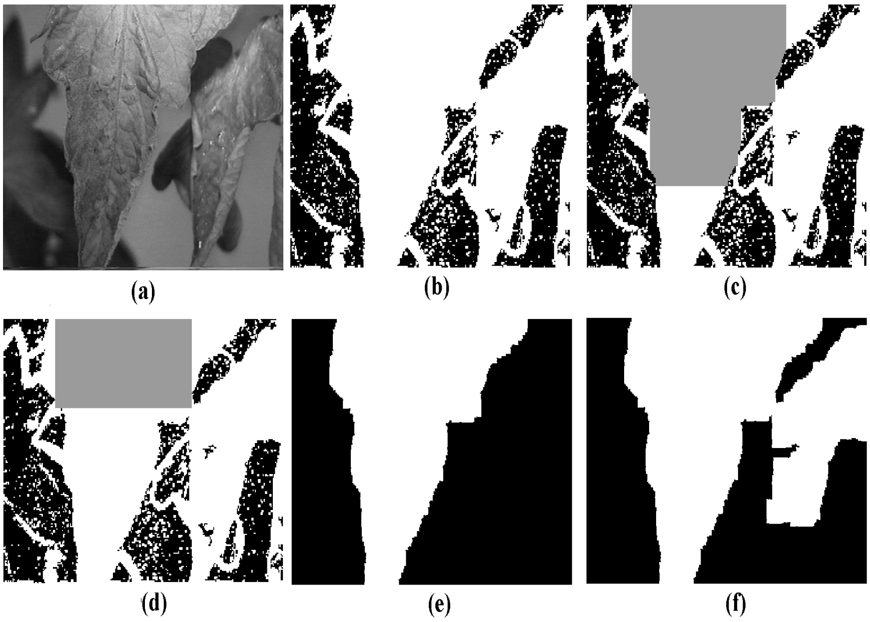

Figure 4. (a) Original image; (b) binary image X; (c) and (d) openings $\gamma_{\mu 1}(X)$ and $\gamma_{\mu 2}(X)$ in gray with $\mu 1 = 82$ and $\mu 2 = 155$, respectively; (e) output image of the openings $\widehat{\gamma}_{\lambda,\mu 1}(X)$ and $\widehat{\gamma}_{\lambda,\mu 2}(X)$ with $\lambda = 18$; (f) output image $\widehat{\gamma}_{\lambda,\mu}(X)$),with $\mu = 81$ and $\lambda = 18$.

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