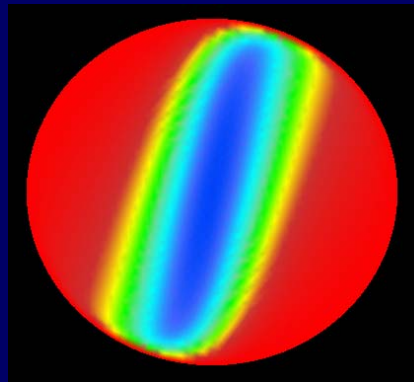


# *Fastflo* computations for fluid-structure interactions

CSIRO Mathematical and Information Sciences  
Clayton, Australia

*Fastflo*



Flexible finite element software for  
the numerical solution of PDEs

# Outline of presentation

- *Fastflo* – summary of features relevant to calculations for fluid dynamics and linear elasticity
- Overview of model equations and algorithms
- 2 examples
  1. fluid flow with time-dependent boundary motions
  2. coupled fluid-elasticity calculations

# Fluid - structure problems: relevant features of *Fastflo*

- able to specify and solve problems in multiple sub-regions
- moving meshes and free surfaces are possible
- can solve systems of PDEs
- able to specify and solve problems on boundaries
- flexible (in terms of geometry, equations, algorithms)
- (almost) any PDE model can be solved
- self-contained (mesh generation, graphics)
- programming environment that empowers users
- very useful for rapid prototyping

# Overview of *Fastflo*

- based on the finite element method, 2D and 3D
- range of element types (linear, quadratic; triangles, quadrilaterals, tetrahedra, hexahedra)
- internal mesh generator for 2D problems
- interface to commercial pre- and post-processors
- includes a high level macro command language to specify and solve PDEs
- graphical user interface

# Overview of *Fastflo* (cont'd)

- selection of sparse matrix solvers (direct and iterative)
- Tutorial Guide, on-line Reference Manual
- many well-documented applications
- incorporates feedback from dozens of licensees
- Fluids ToolBox released with *Fastflo V3*
- available in PC and UNIX versions, both written in C. The PC GUI is built using Borland C++ and makes use of Windows facilities. The UNIX GUI is built using Motif.

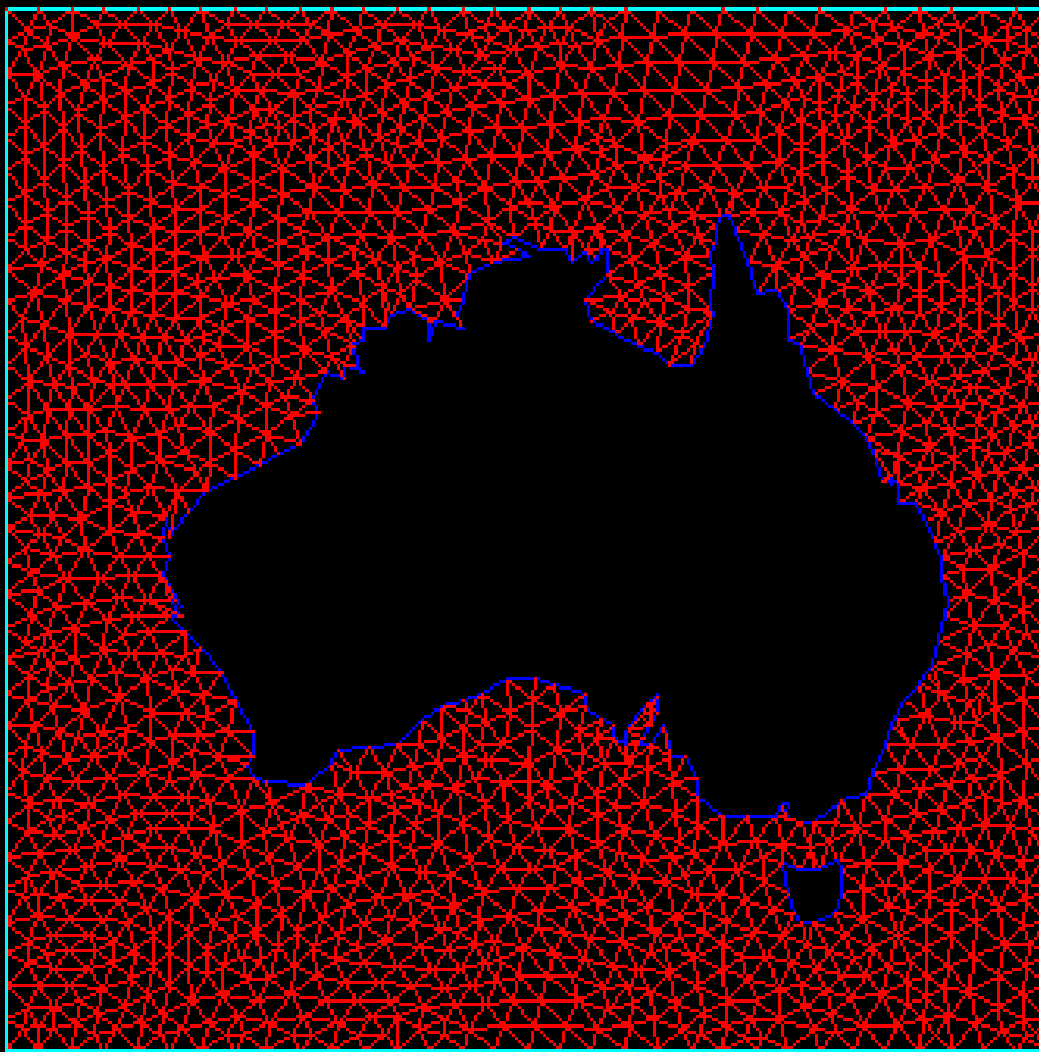
# Design features of *Fastflo*

- users present problems to *Fastflo* via two files:
  - \*.msh which contains geometrical information;
  - \*.prb which contains equations, boundary conditions, the algorithm, and commands to view the results
- data is stored on a vector stack (user-accessible)
- we think of *Fastflo* as a workbench, with tools to specify and solve PDEs; the workbench offers graphics, editing and printing facilities.

# Design features of *Fastflo* (cont'd)

- *Fastflo* macro code is open and portable; there is no need for time-consuming low level programming
- users are free
  - # to specify what equation(s) to solve
  - # to design the algorithm used for the solution
  - # to control the computations intelligently
- substantial guidance is available from an extensive list of examples and extensive documentation
- on-line Help file available for users

# Mesh generation



- \* triangular mesh generator
- \* linear and quadratic approx
- \* 2D: triangles, quadrilaterals
- \* 3D: tetrahedra, hexahedra
- \* can interface to third-party software (especially FEMAP)
- \* isoparametric elements
- \* deformable boundaries
- \* block mesh generator
- \* axisymmetry

# Equations for fluid sub-region

Navier-Stokes equation

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial v_i}{\partial x_j} + \mu \frac{\partial v_j}{\partial x_i} \right)$$

plus incompressibility condition

$$\partial v_j / \partial x_j = 0$$

Note: summation over repeated suffices.

LHS = rate of change of fluid momentum

RHS = nett stress for a Newtonian fluid

# Equations for structural sub-region

Linear elasticity equation:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_j}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \lambda \frac{\partial u_j}{\partial x_j} \right) + F_i$$

LHS = rate of change of momentum (often neglected for linear elasticity, but needs to be retained here).

RHS = combination of nett stress and body forces

(For elasticity,  $\mu$  and  $\lambda$  are the Lamé constants; for fluids  $\mu$  is the viscosity.  $F$  is the body force and  $\rho$  is the density)

# Algorithm for fluid-structure interaction

1. Update velocity and displacements at the start of a timestep.
2. Compute jointly for the velocity in the fluid sub-region and the displacement in the structural sub-region. *Fastflo* ensures that the unknown vector is numerically continuous across the interface. The models must ensure that stress is continuous across the interface.
3. Update the geometry by solving an ALE problem for the new mesh; go to next timestep.

# Algorithm for fluid sub-region

See accompanying file CFD-algorithm.doc. The solver is an intermediate level solver with the following features:

- 2<sup>nd</sup> order Runge-Kutta method for timestepping and handling non-linearities
- pressure and velocity correction in each timestep to ensure incompressibility

# Algorithm for structural sub-region

Let  $l$  denote the timestep. Approximate the LHS of

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_j}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \lambda \frac{\partial u_j}{\partial x_j} \right) + F_i$$

by a central difference expression and the RHS by the average of values at timesteps  $l-1$  and  $l+1$ :

$$\begin{aligned} & \frac{\rho}{(\Delta t)^2} (u_i^{l+1} - 2u_i^l + u_i^{l-1}) \\ &= \frac{1}{2} \left[ \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i^{l+1}}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i^{l+1}}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \lambda \frac{\partial u_j^{l+1}}{\partial x_j} \right) \right. \\ & \quad \left. + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i^{l-1}}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i^{l-1}}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \lambda \frac{\partial u_j^{l-1}}{\partial x_j} \right) \right] \end{aligned}$$

# Structural sub-region (continued)

The expression on the previous slide is a 2<sup>nd</sup> order elliptic equation for the unknown displacement in the structure.

To repeat the key features: (1) the unknown variable is a hybrid of the velocity in the fluid sub-region and the displacement in the structure sub-region; (2) this variable will be automatically continuous across the interface; (3) the equations have been written in such a way that continuity of stress is the natural boundary condition.

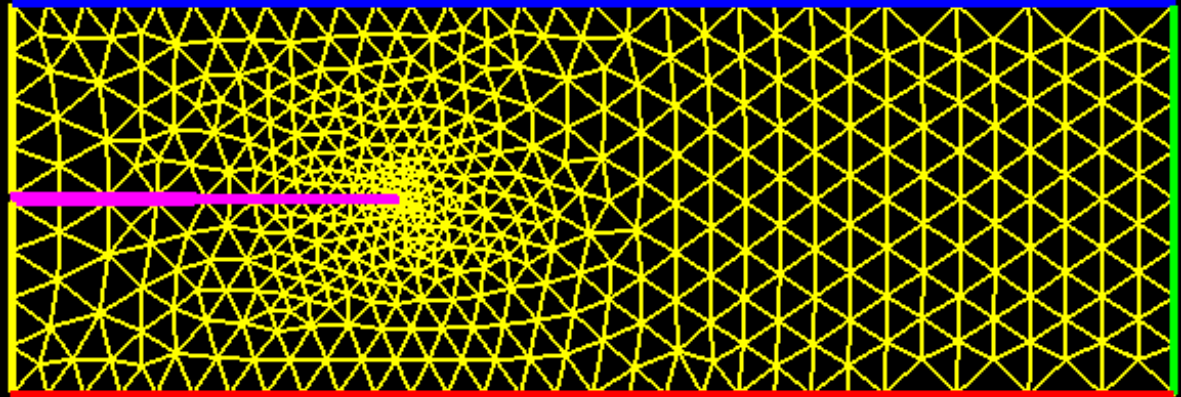
# Mesh movement (ALE method)

See the *Fastflo* Tutorial Guide for a description of the ALE (Arbitrary Lagrangian Eulerian) method. Basically, [Eulerian part] mesh displacements are given by solving an elasticity problem

$$\frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_j}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \lambda \frac{\partial u_j}{\partial x_j} \right) = 0$$

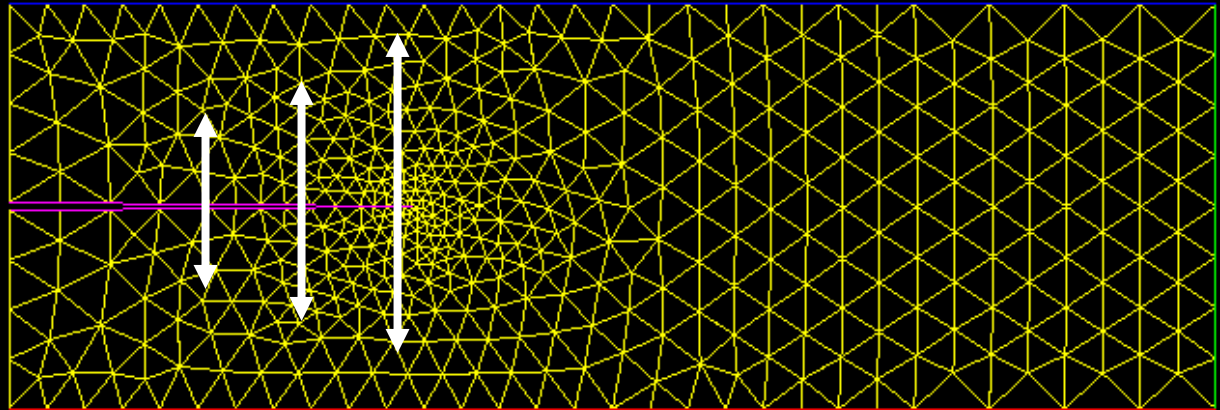
with [Lagrangian part] displacements prescribed at the interface of the moving structure and displacements held zero elsewhere on the boundary.

# Introductory problem: time-dependent boundary motions



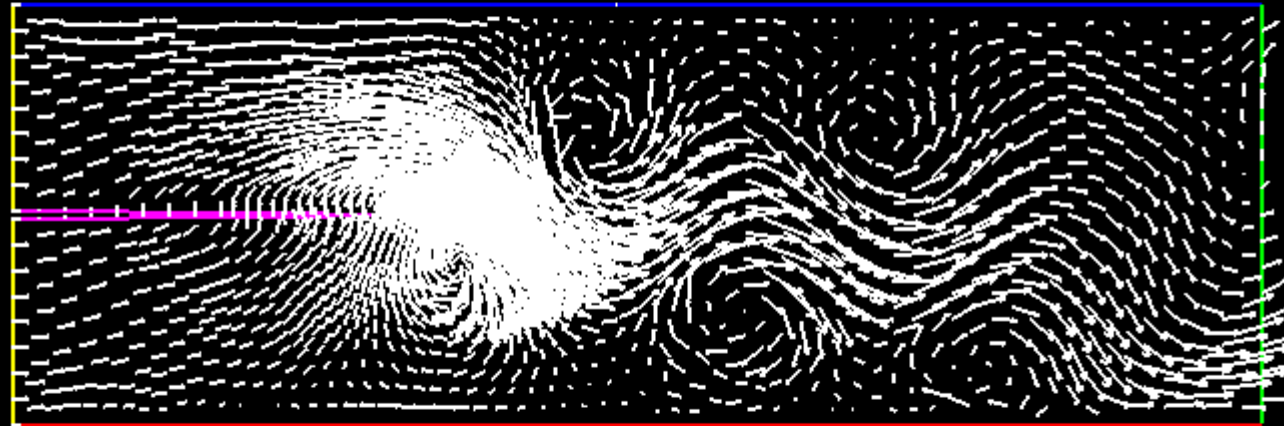
Consider fluid flow from left to right in a 2D duct in which there is a plate that vibrates up and down. The plate is fixed at the LH end. The applied (vertical) vibrational velocity is sinusoidal and increases linearly from zero at the fixed point.

# Introductory problem: time-dependent boundary motions (continued)



Clearly, this is a simplification of the fluid-structure interaction problem - the velocity of the structure is given and there is no need to solve an elasticity problem inside the structural sub-domain

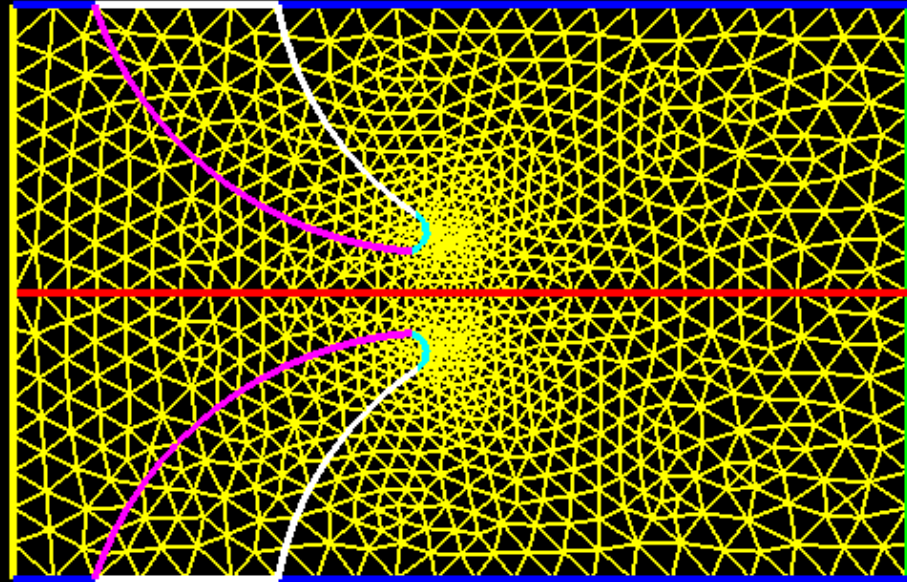
# Time-dependent boundary motions – arrow plot of velocity vector



See the files **kicker.msh** and **kicker.prb**. Results shown are an arrow plot of the velocity vector at 500 timesteps = 5 cycles.

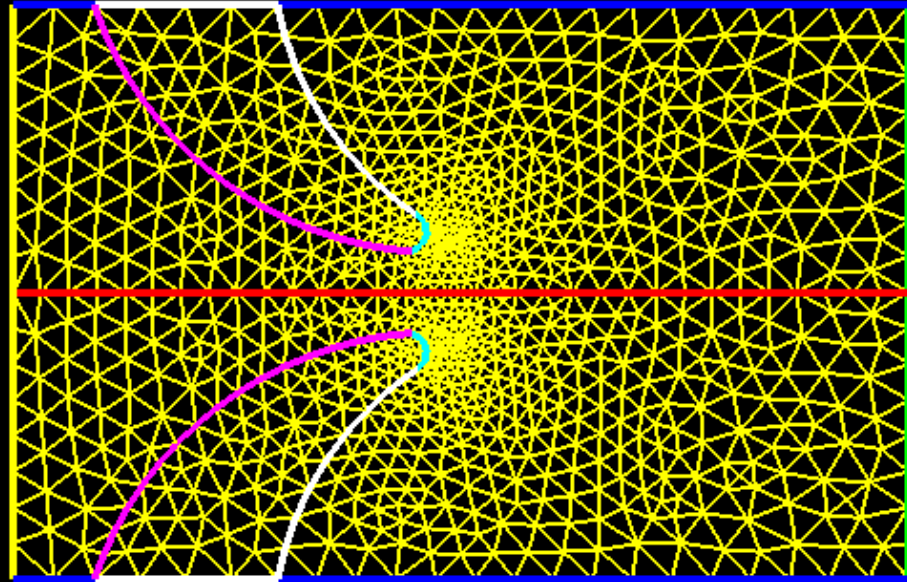
P	Rho	998	% density of water	kg/m <sup>3</sup>
P	Mu	1.002e-3	% viscosity of water	kg/(m.s)
P	Vflow	0.05	% injection speed	m/s
P	Lplate	0.01	% length of plate	m
P	width	0.01	% width of duct	m
P	Hertz	10	% vibrations/second	s <sup>(-1)</sup>
P	amplit	0.001	% amplitude of vibration	m
P	deltaT	0.001	% timestep	s

# Main problem: flow through a valve



Consider fluid flow from left to right in a 2D duct in which there is an elastic valve. Computation is made only in the half-space, with a symmetry condition at the centreline.

# Flow through a valve (continued)



Algorithm: as explained earlier. We solve for a hybrid variable, which is the fluid velocity in region 1 and elastic displacement in region 2. The files are given in **valve.msh** and **valve.prb**.

See also CFD-algorithm.doc

# Flow through a valve (parameters)

## % fluid and physical data

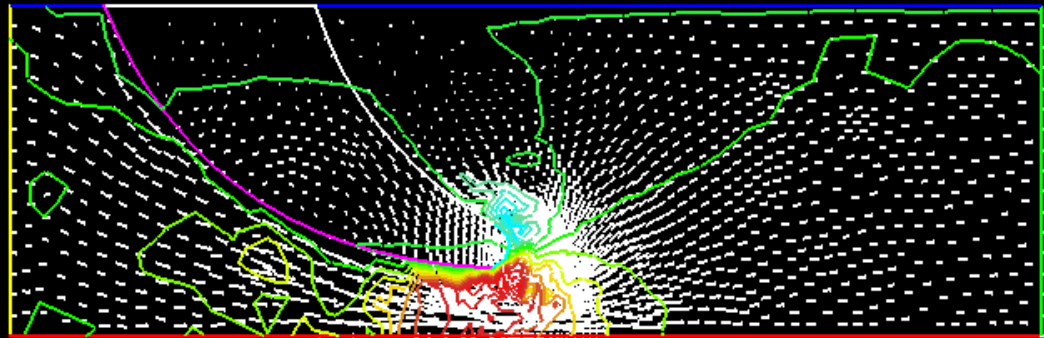
P	Rho	998	% density of water	kg/m <sup>3</sup>
P	Mu	1.002e-3	% viscosity of water	kg/(m.s)
P	Vflow	0.2	% injection speed	m/s
P	width	0.007	% half-width of duct	m
% computational control parameters				
P	deltaT	0.0001	% timestep	s
P	STOPsteps	500	% maximum number of timesteps	
P	SteadyTest	0.001/deltaT	% convergence test on timestepping	
P	Modulus	1e5	% Young's modulus	Pa
P	Ratio	0.45	% Poisson Ratio	dimensionless

Mesh generated by *Fastflo*'s unstructured mesh generator, with concentration near the tip of the valve: 1922 nodes, 923 six-noded triangles. Computation time for 100 timesteps: 763 secs on a 500 MHz PC.

# Flow through a valve: results at 50 timesteps = 0.005 sec

50 timesteps: An elastic wave has passed down to the end of the valve, which is still shuddering elastically. The valve has opened by about 45%. No flow separation has occurred.

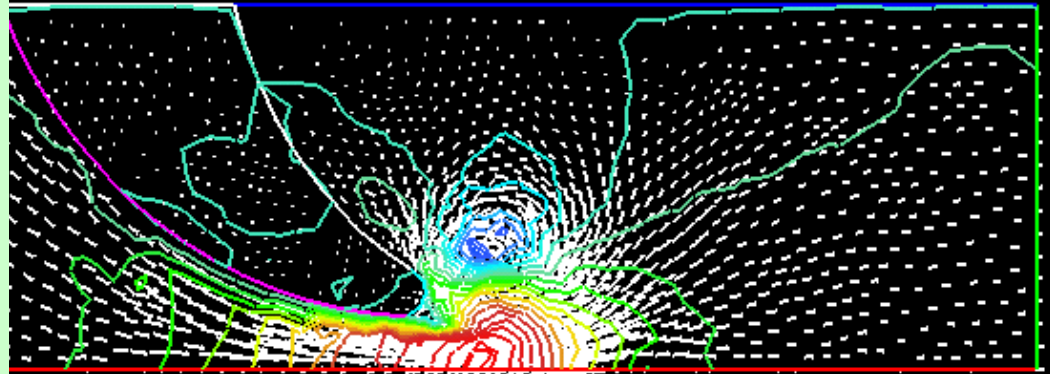
The graphic display shows an arrow plot of velocity vector and a contour plot of V201



# Flow through a valve: results at 100 timesteps = 0.01 sec

100 timesteps: Separated flow has just occurred. This causes a hydrodynamic stress that recloses the valve somewhat. Momentarily the valve is steady in this snapshot.

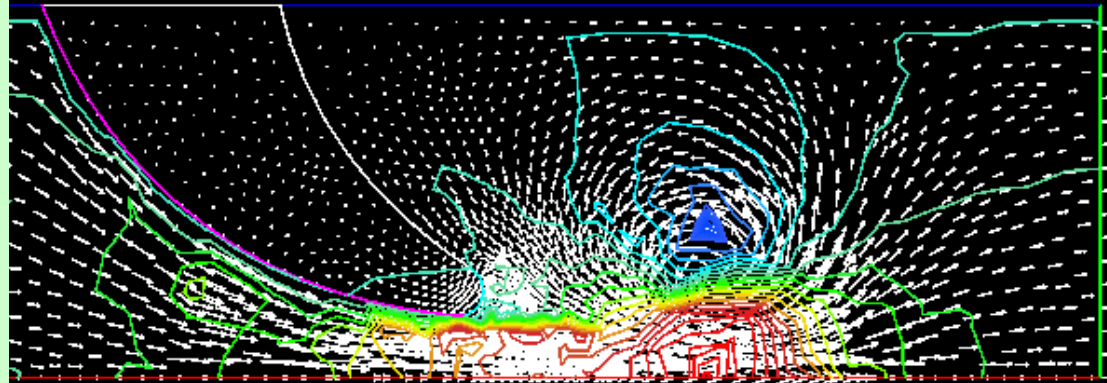
The graphic display shows an arrow plot of velocity vector and a contour plot of V201



# Flow through a valve: results at 150 timesteps = 0.015 sec

150 timesteps: The valve is fully open, although the tip is still shuddering elastically. A large flow separation region has formed downstream of the valve. Further vortex shedding is expected.

The graphic display shows an arrow plot of velocity vector and a contour plot of V201



# Flow through a valve - discussion

- This algorithm is presented as a demonstrator.
- Relatively small timesteps are required to resolve the motions, both elastic and fluid, as well as the coupling.
- We solve for a hybrid variable (velocity in fluid, displacement in solid); with further attention to the algorithm, we expect to produce a model in which the unknown variable is uniformly a velocity.

# Discussion (continued)

- The use of linear elasticity for the valve is valid for a small range of displacements with particular materials. For biological materials, we would need a more sophisticated model, perhaps anisotropic, perhaps flexible but inextensible.
- The fluid solver can be replaced by a more sophisticated solver (operator-splitting).

# Discussion (continued)

- For this multi-region calculation, we make a joint solution in regions 1 and 2. It is currently possible (but slower) to use a model with two stages. In the liquid stage – the flow equations are solved in region 1 and a dummy problem in region 2. In the solid stage – the elasticity equations are solved in region 2 and a dummy problem in region 1. Coupling must be carefully modelled.
- In the near future, we will release a version of *Fastflo* with enhanced multi-region capability. Dummy problems will not be required.

# Summary of presentation

- We summarised the features of *Fastflo* that are appropriate for fluid-structure interaction problems. We also summarised the design features of *Fastflo*.
- We described general models and algorithms for addressing laminar incompressible flow around elastic structures.
- We solved two examples: (1) flow past a moving boundary, (2) flow through an elastic valve.